

Sec. 5.4 Dividing Polynomials

Divide using long division

$$(3x^3 + 9x^2 + 8x + 4) \div (x + 2)$$

$$\begin{array}{r} 3x^2 + 3x + 2 \\ -24 + 36 - 16 + 4 \\ \hline 12 - 12 \\ \hline 0 \end{array}$$

$$\underline{\hspace{10em}} \quad 0 \quad f(-2) = 0$$

Ex:  $(5x^2 + 2x + 3) \div (x + 1) \quad \frac{22}{7} = 3\frac{1}{7}$

$$\begin{array}{r} 5x - 3 + \frac{6}{x+1} \text{ Remainder} \\ \hline \text{Divisor} \\ \hline 5x \quad (x+1) \quad \underline{5x^2 + 2x + 3} \\ -5x^2 + 5x \\ \hline -3x + 3 \\ +3x + 3 \\ \hline 6 \end{array}$$

NOT a factor  $\leftarrow$   $\boxed{6}$   $\rightarrow f(-1) = 6$

$\textcircled{7} \overline{)22} \begin{array}{r} 3 \\ -21 \\ \hline 1 \end{array}$

Ex: Is  $x^2 - 1$  a factor of  $3x^4 - 4x^3 + 12x^2 + 5$ ?

$$\begin{array}{r} 3x^2 - 4x + 15 \\ \hline 3x^2 \quad (x^2 + 0x - 1) \quad \underline{3x^4 - 4x^3 + 12x^2 + 0x + 5} \\ -3x^4 + 0x^3 + 3x^2 \\ \hline -4x^3 + 15x^2 + 0x \\ +4x^3 + 0x^2 + 4x \\ \hline 15x^2 - 4x + 5 \\ -15x^2 + 0x + 15 \\ \hline -4x + 20 \end{array}$$

$\boxed{3x^2 - 4x + 15 + \frac{-4x + 20}{x^2 - 1}}$

Divide polynomial  $P(x)$  of degree  $n \geq 1$   
by  $x - a$ , then the remainder is  $P(a)$ .

$$(x^3 - 3x^2 - 5x - 25) \div (x - 5)$$

$$\begin{array}{r}
 \phantom{x^3} x^2 + 2x + 5 \quad \begin{array}{l} 125 - 75 - 25 - 25 \\ 0 \end{array} \\
 \hline
 \begin{array}{l} x^3 - 3x^2 - 5x - 25 \\ -x^3 + 5x^2 \\ \hline 2x^2 - 5x \\ -2x^2 + 10x \\ \hline 5x - 25 \\ -5x + 25 \\ \hline 0 \end{array}
 \end{array}$$

CHANGE SIGNS

$x - 5$  and  $x^2 + 2x + 5$  are factors of  $x^3 - 3x^2 - 5x - 25$ .  
 Remainder Theorem:  $f(5) = 0$   
 $f(a) = R$

1  
 (1)  $\frac{a}{2}$   
 R 1

Synthetic Division (Substitution)

a.  $(5x^2 + 2x + 3) \div (x + 1) = 0$   
 $x = -1$

$\begin{array}{r|rr} -1 & 2 & 3 \\ & -5 & \\ \hline & 5x & -3 \end{array}$  Coefficients  
 mult.  $\rightarrow 5x - 3 \mid 6R. \rightarrow f(-1) = 6$   
 $5x - 3 + \frac{6}{x+1}$

b.  $(x^4 + 3x^3 + 7x^2 + 26x + 15) \div (x + 3)$

$\begin{array}{r|rrrrr} -3 & 1 & 3 & 7 & 26 & 15 \\ & -3 & 0 & -21 & -15 & \\ \hline & 1x^3 & 0 & 7 & 5 & 0 \end{array}$   $f(-3) = 0$   
 $x + 3 \rightarrow$  factors  
 $x^3 + 7x + 5$

c.  $(x^4 - 5x^2 + 4x + 12) \div (x + 2)$   
 $(x^4 + 0x^3 - 5x^2 + 4x + 12)$

$\begin{array}{r|rrrrr} -2 & 1 & 0 & -5 & 4 & 12 \\ & -2 & 4 & 2 & -12 & \\ \hline & 1 & -2 & -1 & 6 & 0 \end{array}$   $f(-2) = 0$   
 $x^3 - 2x^2 - x + 6$  factors  
 $x + 2$

d.  $(x^4 - \frac{9}{2}x^3 + 3x^2 - \frac{1}{2}x) \div (x - \frac{1}{2})$

$\begin{array}{r|rrrrr} \frac{1}{2} & 1 & -\frac{9}{2} & 3 & -\frac{1}{2} & 0 \\ & \frac{1}{2} & -2 & \frac{1}{2} & 0 & \\ \hline & 1 & -4 & 1 & 0 & 0 \end{array}$   $f(\frac{1}{2}) = 0$   
 $x^3 - 4x^2 + x$  factors  
 $x - \frac{1}{2}$

$(x-1)(x^3 - 4x^2 + x)$  QE  
 $(x-1)(x)(x^2 - 4x + 1) = 0$  -Factor X  
-CTS ✓  
-QF ✓

$x = 1 \quad x = 0$   $x^2 - 4x + 1 = 0$   
 $x^2 - 4x + 4 = -1 + 4$   
 $(x - 2)^2 = 3$   
 $x - 2 = \pm\sqrt{3}$   
 $\quad \quad \quad +2 \quad \quad +2$   
 $x = 2 \pm \sqrt{3}$

