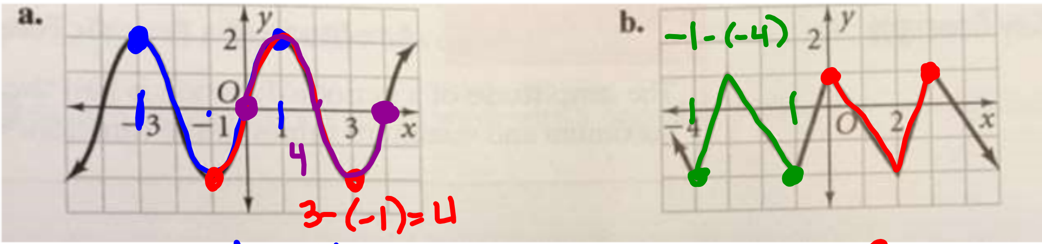
13.1 Exploring Periodic Data

Periodic function - repeats a pattern of y-vales (outputs) at regular intervals.
Cycle - 1 complete pattern. A cycle may begin at any point on the graph of the function.
Period - the horizontal length of 1 cycle, - in terms of x-values.

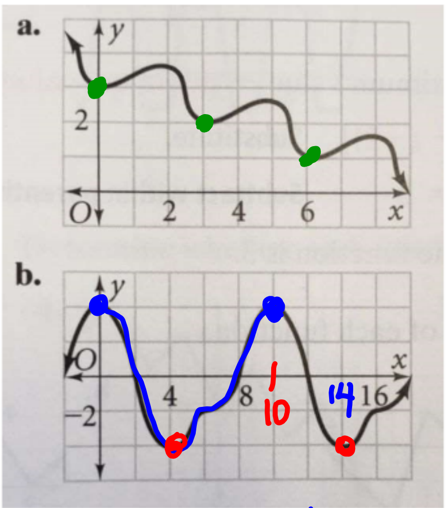
Example 1: Identifying Cycles and Periods
Analyze the periodic function. a) Identify 1 cycle in two different ways. b) Determine the period of the function.



period : 4
1-(-3)=4

p : 3

Example 2: Identifying Periodic Functions
Determine whether each function is or is not periodic. If it is, find the period.



not periodic
y-values are not repeated

period : 10

14-4

* $\frac{max - min}{2}$ = amplitude

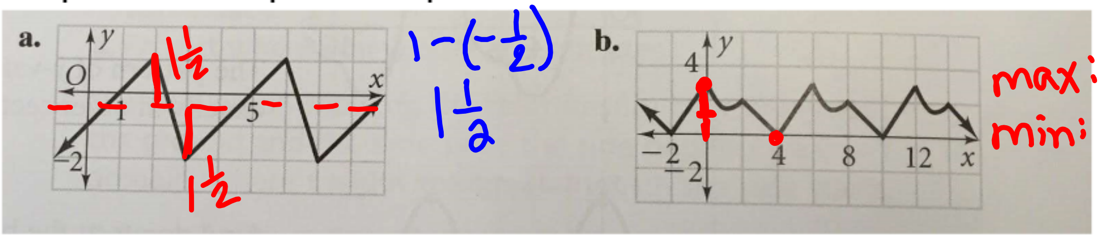
Amplitude - of a periodic function measures the amount of variation in the y-values. To find the amplitude:

① Find average of the max + min = MIDLINE

$\frac{max + min}{2} = y$ ② max − midline = amplitude

min − midline = amplitude

Example 3: Find the Amplitude of the periodic function.



$1 - (-\frac{1}{2})$

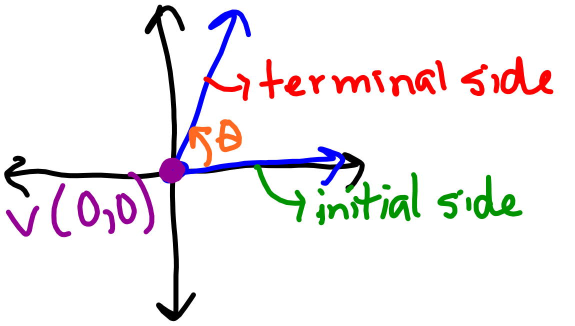
$1\frac{1}{2}$

max: 3

min: 0

$\frac{1 + -2}{2} = -\frac{1}{2} = y$

$\frac{3-0}{2} = \frac{3}{2} = a$



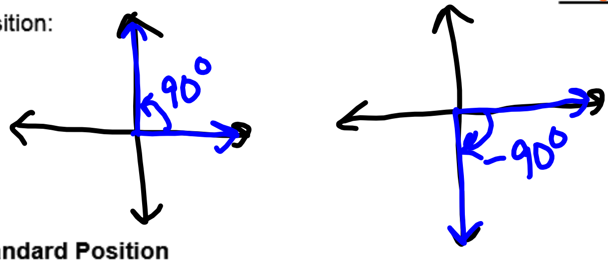
13.2 Angles (day 1)

An **angle in standard position** has:

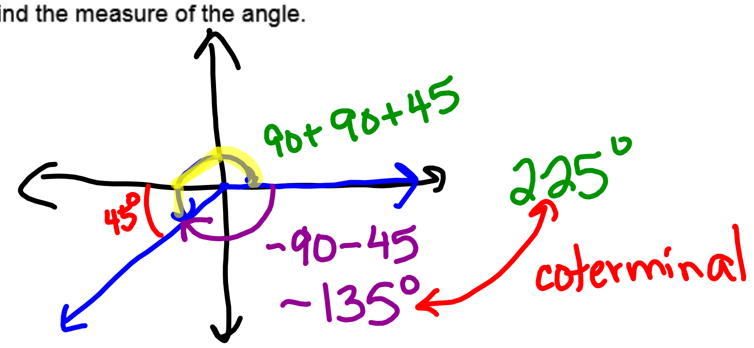
- vertex is at the (0,0)
- one ray is on the x-axis

Initial side - the ray on the positive x-axis
Terminal side - the other ray of the angle
The **measure of an angle in standard position** is the amount of rotation from the initial side to the terminal side.
The measure of an angle is **positive** when the rotation from the initial side to terminal side is counterclockwise.
The measure of an angle is **negative** when the rotation from the initial side to terminal side is clockwise

Sketch and angle in standard position:

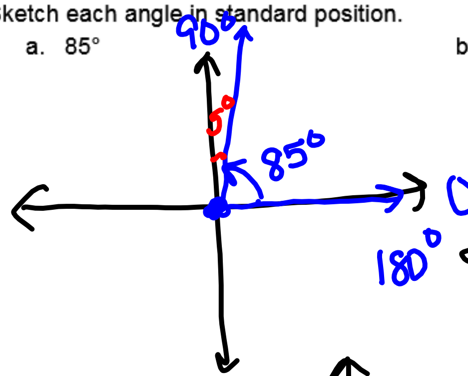


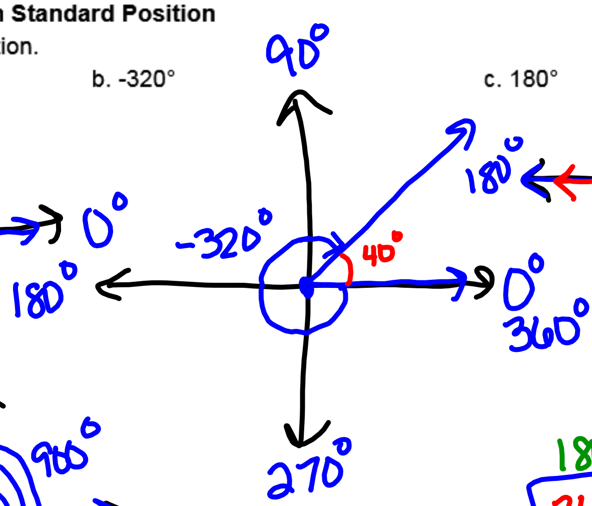
Example 1: Measuring an Angle in Standard Position
Find the measure of the angle.

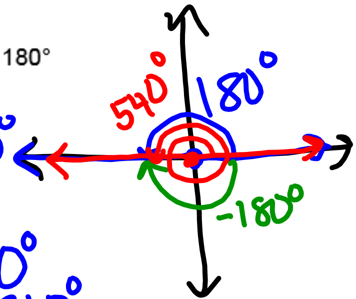


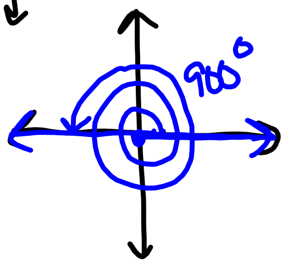
Example 2: Sketching an Angle in Standard Position
Sketch each angle in standard position.

a. 85° b. -320° c. 180°









180 − 360 = −180
360 + 180 = 540
+ 360
900°

Coterminal Angles - Two angles in standard position are coterminal if they have the same terminal side. For any given angle, there are infinitely many coterminal angles. To find a coterminal angle, **add or subtract 360°.**

Example 3: Finding Coterminal Angles

a. Find a positive angle and a negative angle that are coterminal with 198°.

$198 + 360 = 558°$

$198 - 360 = -162°$

b. Are the angles with measure 40° and 680° coterminal? Explain.

$680 - 360 = 320° - 360° = -40°$

not coterminal

$680 - 40 = 640 \div 360 = X$

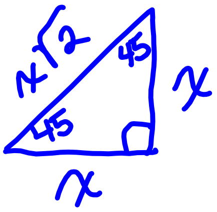
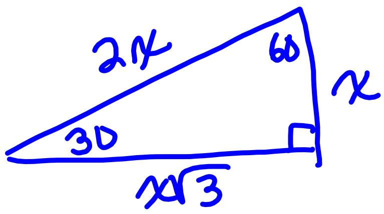
c. Find the measure of an angle between 0° and 360° coterminal with 385°.

$385° - 360° = 25°$

d. Find the measure of an angle between 0° and 360° coterminal with -356°.

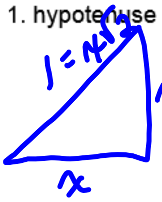
$-356° + 360° = 4°$

Recall: Special Right Triangles

45°-45°-90° Let a leg be x.	30°-60°-90° Let the shorter leg be x.
	

Find the missing side lengths in each 45°-45°-90° triangle. Rationalize any denominators.

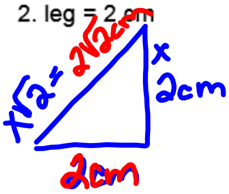
1. hypotenuse = 1 inch



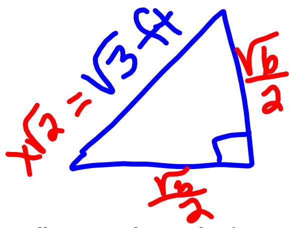
$\frac{\sqrt2}{\sqrt2}\cdot\frac{1}{\sqrt2} = \frac{x\sqrt2}{\sqrt2}$

$\frac{\sqrt2}{2} = x$

2. leg = 2 cm



3. hypotenuse = √3 ft



$\frac{x\sqrt2}{\sqrt2} = \frac{\sqrt3}{\sqrt2}$

$x = \frac{\sqrt3}{\sqrt2}\cdot\frac{\sqrt2}{\sqrt2}$

$x = \frac{\sqrt6}{2}$

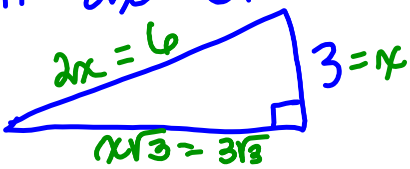
Find the missing side lengths in each 30°-60°-90° triangle. Rationalize any denominators.

4. shorter leg = 3 inch

SL $x = 3$

LL $x\sqrt3 = 3\sqrt3$ in

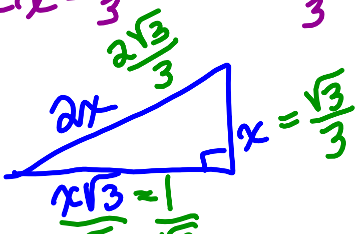
H $2x = 6$ in



5. longer leg = 1 cm

$\frac{x\sqrt3}{\sqrt3} = \frac{1}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3}$

SL $x = \frac{\sqrt3}{3}$ LL $= \frac{2\sqrt3}{3}$



$\frac{x\sqrt3}{\sqrt3} = \frac{1}{\sqrt3}$

$x = \frac{1}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3} = \frac{\sqrt3}{3}$

6. hypotenuse = 1 ft

$\frac{2x}{2} = \frac{1}{2}$

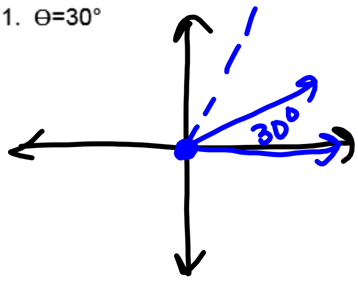
SL $x = \frac{1}{2}$

LL $x\sqrt3 = \frac{1}{2}\cdot\sqrt3 = \frac{\sqrt3}{2}$

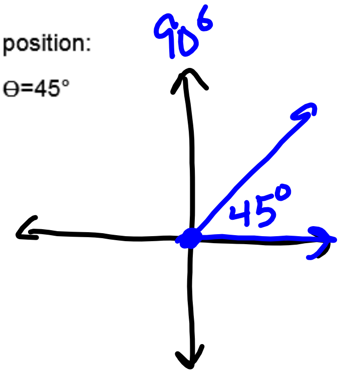
13.2 The Unit Circle (day 2)

Warm Up: Sketch the angle in standard position:

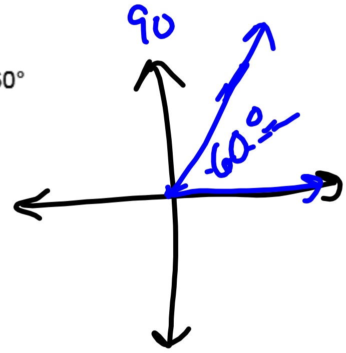
1. θ=30°



2. θ=45°



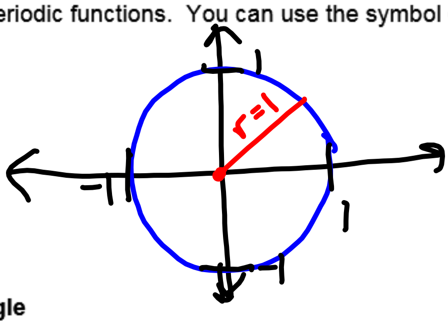
3. θ=60°



Unit Circle - a circle with a radius of 1 and its center is at the vertex

Points on the unit circle are related to periodic functions. You can use the symbol θ "theta" for the measure of an angle in standard position.

Sketch the unit circle:



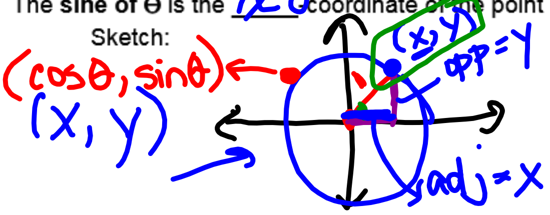
Definition: Cosine and Sine of an Angle

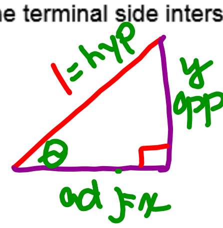
Given: an angle in standard position with measure θ.

The **cosine of θ** is the y -coordinate of the point at which the terminal side intersects the unit circle.

The **sine of θ** is the x coordinate of the point at which the terminal side intersects the unit circle.

Sketch:





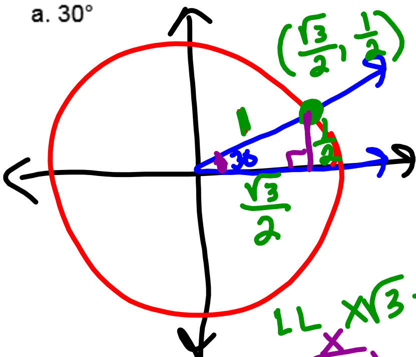
SOHCAHTOA

$\sin\theta = \boxed{\frac{opp}{hyp}} = \boxed{\frac{y}{1}} = \boxed{y}$

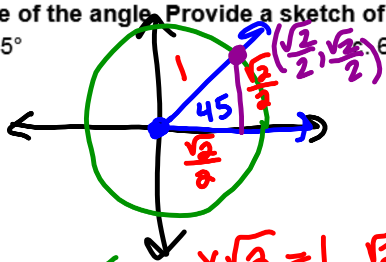
$\cos\theta = \boxed{\frac{adj}{hyp}} = \frac{x}{1} = \boxed{x}$

Example 4&5: Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

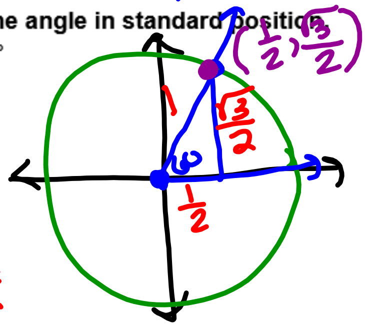
a. 30°



b. 45°



c. 60°



$x\sqrt{3} = \frac{1}{2}\cdot\sqrt{3} = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2} = \boxed{\frac{adj}{hyp}} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$

$\sin 30^\circ = \frac{1}{2} = \boxed{\frac{opp}{hyp}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

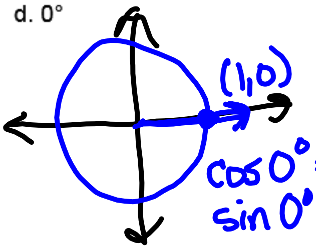
$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

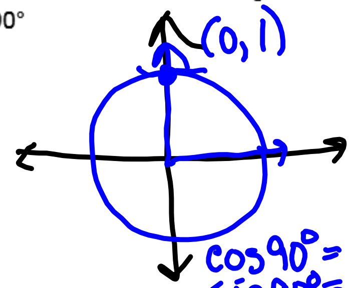
d. 0°



$\cos 0° = 1$

$\sin 0° = 0$

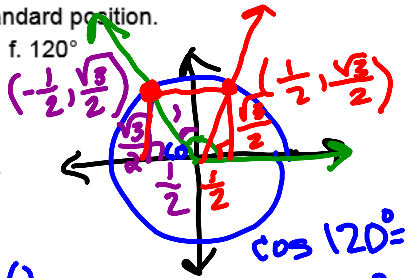
e. 90°



$\cos 90° = 0$

$\sin 90° = 1$

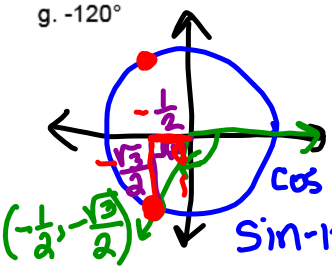
f. 120°



$\cos 120° = -\frac{1}{2}$

$\sin 120° = \frac{\sqrt{3}}{2}$

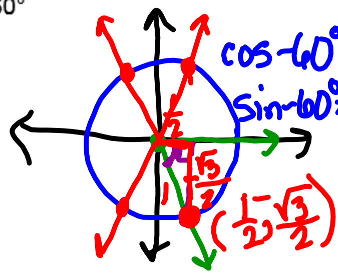
g. -120°



$\cos -120° = -\frac{1}{2}$

$\sin -120° = -\frac{\sqrt{3}}{2}$

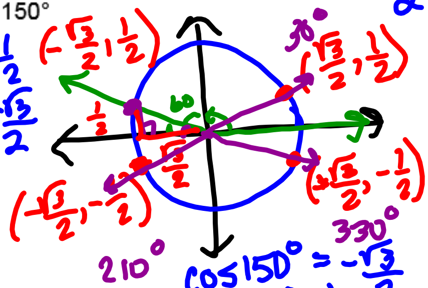
h. -60°



$\cos -60° = \frac{1}{2}$

$\sin -60° = -\frac{\sqrt{3}}{2}$

i. 150°



$\cos 150° = -\frac{\sqrt{3}}{2}$

$\sin 150° = \frac{1}{2}$

Practice Problems:

Calculator Needed: For angles that are not a multiple of 30° or 45°, you will need your calculator. Find cosΘ and sinΘ.

1. Θ=32°

$\cos 32° = 0.848$

$\sin 32° = 0.530$

2. Θ=-210°

$\cos -210° = -0.866$

$\sin -210° = 0.5$

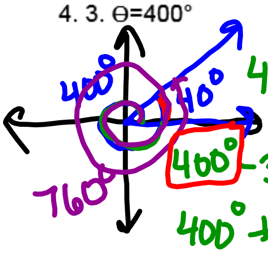
3. Θ=-10°

$\cos -10° = 0.984$

$\sin -10° = -0.174$

Find a positive and negative coterminal angle for the given angle.

4. 3. Θ=400°



$40 - 360 = \boxed{-320°}$

$\boxed{400°} - 360° = \boxed{40°}$

$400° + 360° = \boxed{760°}$

5. Θ=-125°

$-125° + 360° = 235°$

$-125° - 360 = -485°$

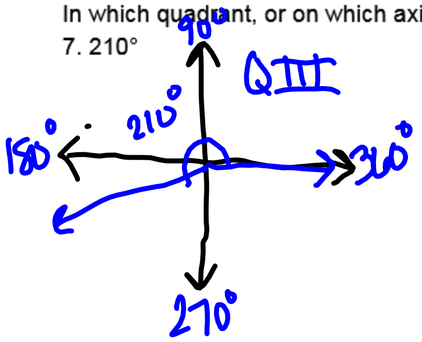
6. 3. Θ=-57°

$-57° + 360° = 303°$

$-57 - 360° = 417°$

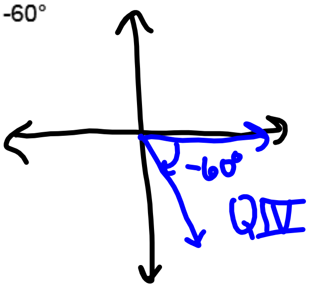
In which quadrant, or on which axis, does the terminal side of each angle lie? Sketch the angle to help you.

7. 210°



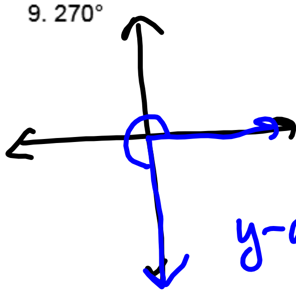
QIII

8. -60°



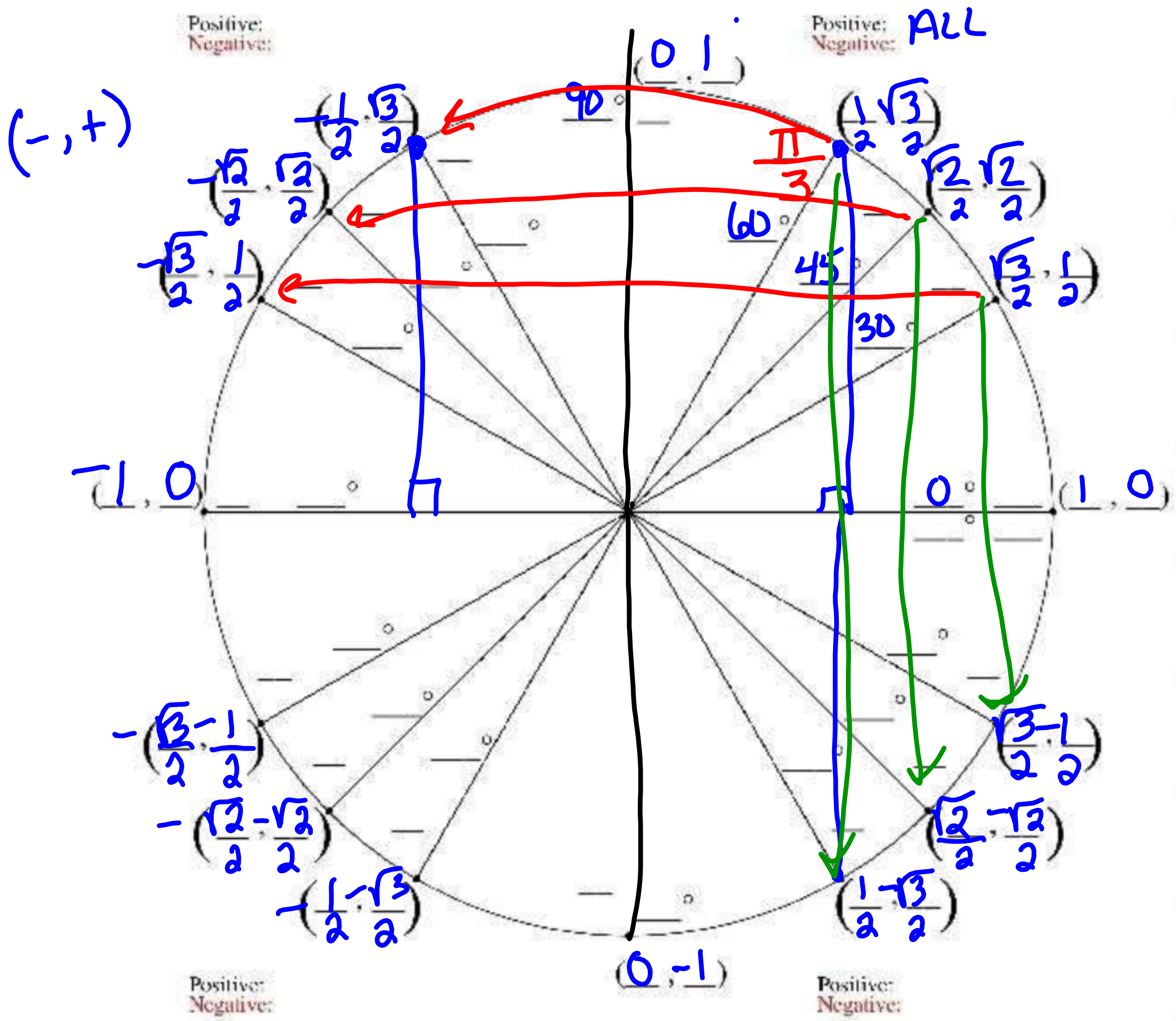
QIV

9. 270°



y-axis

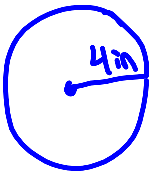
Fill in The Unit Circle



13.3 Radian Measure (Day 1)

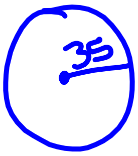
Warm Up: Find the circumference of a circle with the given radius or diameter. Round your answer to the nearest tenth.

1. radius 4 in



$C = 2\pi r$
$= 2\pi \cdot 4$
$= 8\pi$ in
$= 25.1$ in.

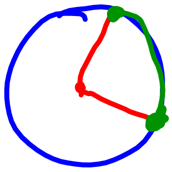
2. diameter 70 m



$C = 2\pi r$
$= 2\pi \cdot 35$
$= 70\pi$ m
$= 219.9$m

Central angle - an angle whose vertex is the center of a circle

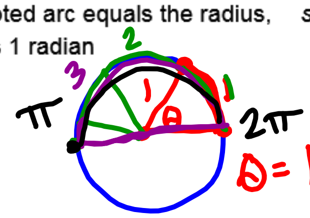
sketch:



$\boxed{\textbf{Intercepted arc}}$ - the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

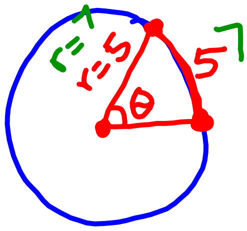
Radian - when the intercepted arc equals the radius, the measure of the angle is 1 radian

$C = 2\pi \cdot 1$
$= 2\pi$



$\theta = 1$ radian

sketch:



$\theta = 1$ radian

- The circumference of a circle is $2\pi r$. Thus there are 2π radians in any circle.
- Since 2π radians = 360°, then π radians = 180°
- Thus you can use this proportion to convert between degrees and radians. $180° = \pi$ radians

D→R: $\dfrac{\pi \text{ rad.}}{180°}$ R→D: $\dfrac{180°}{\pi \text{ rad.}}$

Example 1: Use a proportion

a. Find the radian measure of 60°.

$\cancel{60}° \cdot \dfrac{\pi \text{ rad}}{\cancel{180°}_{3}} = \dfrac{\pi}{3}$ rad.

b. Find the degree measure of $\frac{5\pi}{2}$ *radians*

$\dfrac{5\cancel{\pi}}{\cancel{2}} \cdot \dfrac{\cancel{180°}^{90}}{\cancel{\pi}} = 450°$

Converting between Radians and Degrees

- To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$ rad
- To convert radians to degress, multiply by $\frac{180}{\pi}$

Example 2: Using Dimensional Analysis

Convert the angle to degrees. Round to the nearest degree.

a. $-\frac{3\pi}{4}$ *radians*

$-\frac{3\cancel{\pi}}{\cancel{4}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}} = -135^\circ$

b. $\boxed{\frac{\pi}{2}}$ *radians*

$\frac{\cancel{\pi}}{2} \cdot \frac{180}{\cancel{\pi}}$

90°

c. 2 *radians*

$2\cancel{\text{rad}} \cdot \frac{180^\circ}{\pi\ \cancel{\text{rad}}}$

$\frac{360^\circ}{\pi}$

Convert the angle to radians. Round to the nearest hundredth.

d. 27°

$\cancel{27}^{3} \cdot \frac{\pi}{\cancel{180}_{20}} = \frac{3\pi}{20}$

$= 0.47$

e. 225°

$\cancel{225^\circ}^{\cancel{45}\ 5} \cdot \frac{\pi}{\cancel{180^\circ}_{\cancel{36}\ 4}}$

$\frac{5\pi}{4}$ 3.93

f. 150°

Example 3: Find the exact values of cosΘ and sinΘ for each angle measure.

Step 1: Convert to degrees.

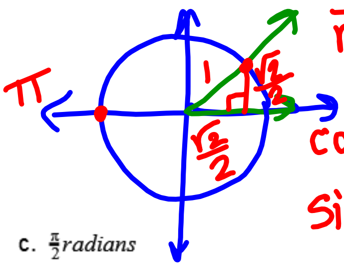
Step 2: Draw the angle. The terminal side is the hypotenuse.

Step 3: Complete the right triangle. Draw a leg to the ***x-axis***.

Step 4: State the cosΘ and sinΘ.

a. $\frac{\pi}{4}$ *radians*

$\frac{\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}} = 45^\circ$



$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

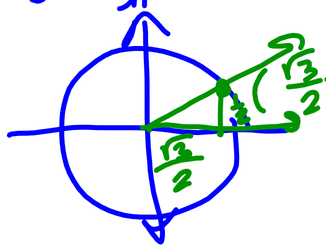
$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

b. $\frac{\pi}{6}$ *radians*

$\frac{\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}} = 30^\circ$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$



$\sin\frac{\pi}{6} = \frac{1}{2}$

c. $\frac{\pi}{2}$ *radians*

d. $\frac{5\pi}{6}$ *radians*

13.3 Arclength (Day 2)

Warm Up: Convert the angle to degrees. Draw the angle in standard position, as well as the corresponding right triangle. Then find the exact values for cosΘ and sinΘ.

$\theta = \frac{\pi}{3}$

$$\frac{\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{\,60}}{\cancel{\pi}} = 60^\circ$$

You can find the length of an intercepted arc by using the proportion:

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure Θ (in *radians*), the length *s* of the intercepted arc is:

$$s = r \cdot \theta$$

Example 4: Finding the Length of an Arc

Find the length of the intercepted arc to the nearest tenth. Sketch a diagram!

a. Given: A circle of radius 3 in, $\theta = \frac{5\pi}{6}$.

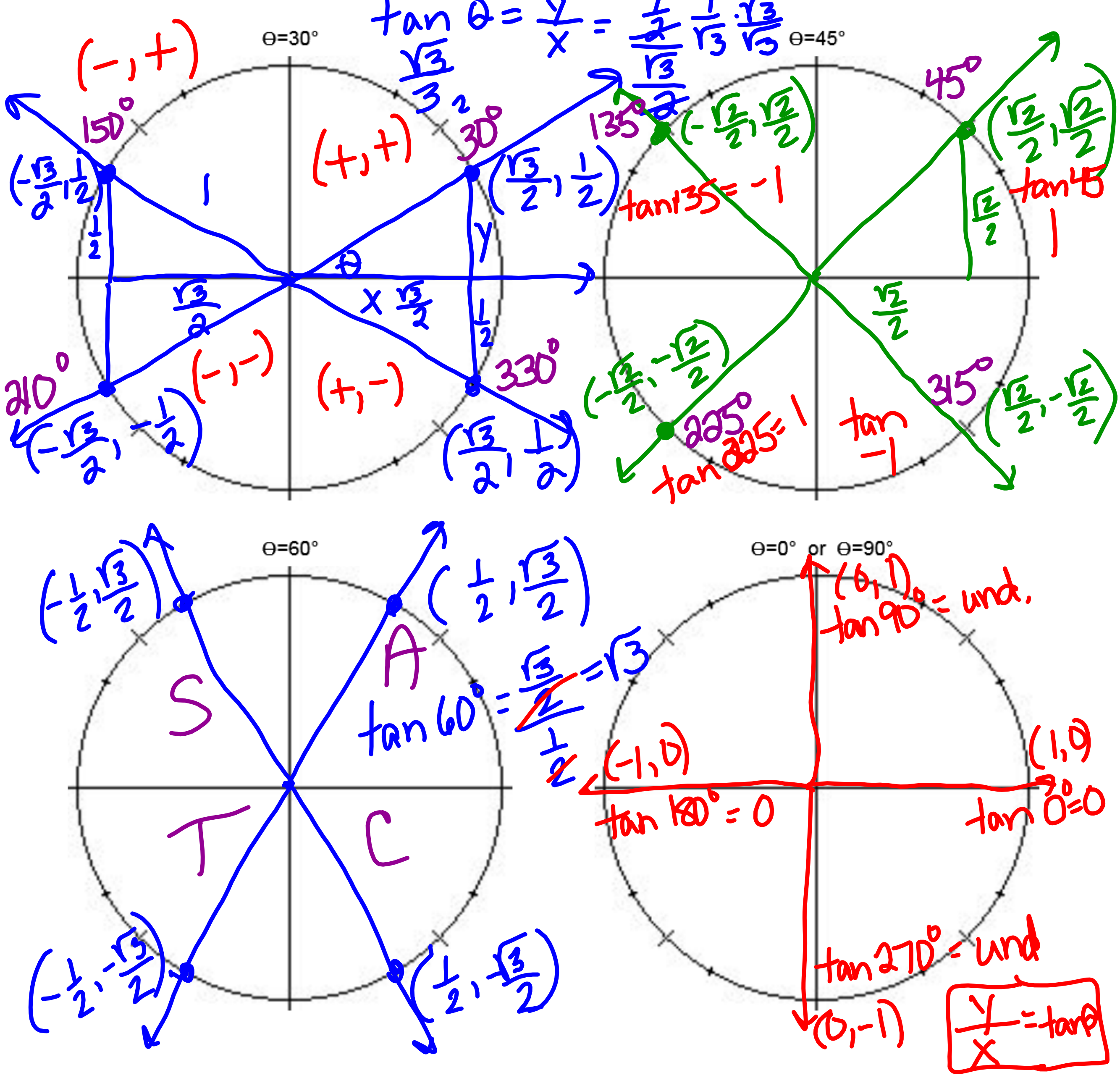
$$s = \frac{5\pi}{\cancel{6}_2} \cdot \cancel{3}^{\,1} \quad \frac{5\pi}{2} \text{ in}$$

b. Given: A circle of radius 5m, $\theta = \frac{2\pi}{3}$.

$$\overset{\theta}{\frac{\pi}{6}} \cdot \overset{r}{10}$$

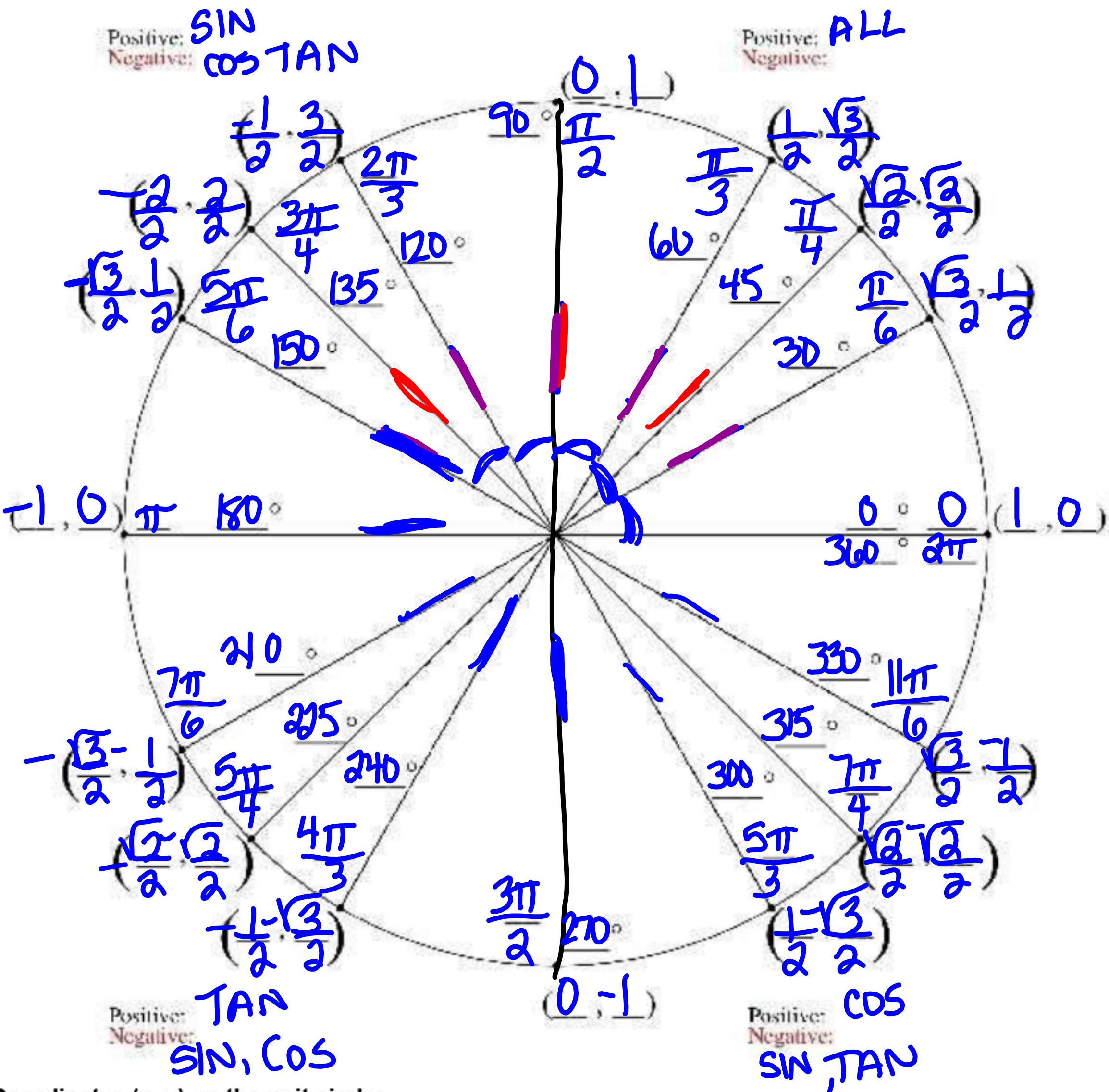
The Unit Circle: radius = ________

If you know quadrant 1, you can derive quadrants 2, 3, 4 by symmetry. Thus, let's study quadrant 1.



Now let's do all four quadrants...

Fill in The Unit Circle



Coordinates (x, y) on the unit circle:

cosΘ = x

sinΘ = y

tanΘ = $\frac{y}{x}$

13.4 The Sine Function (Day 1)

[→ included
(→ not included

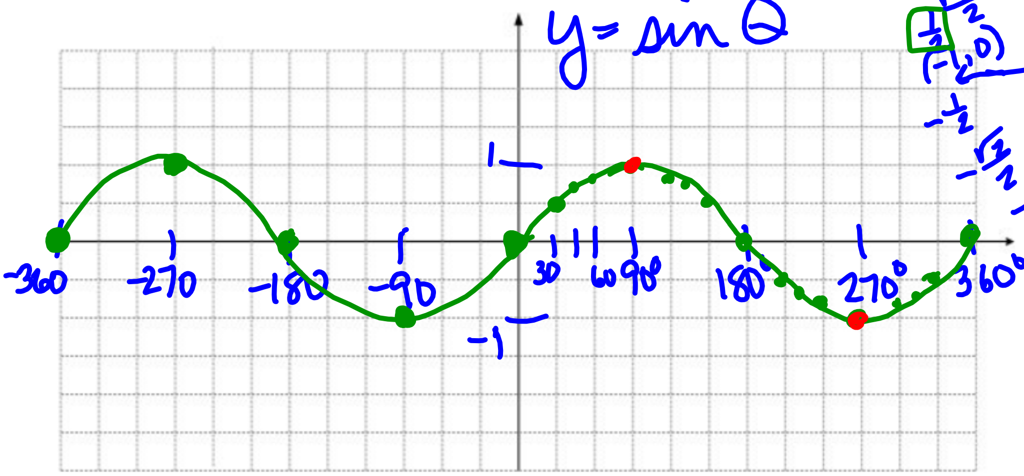
Warm Up: Use the graph. State:

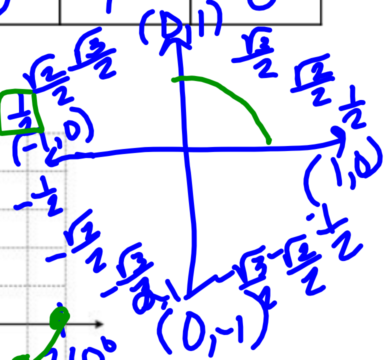
1. the period 2π
2. the domain ARN
3. the amplitude a
4. the range $[-1, 1]$ $\quad -1 \le y \le 1$

sine function y=sinΘ: for each measure of Θ, the sine of Θ corresponds with the ______-coordinates on the unit circle.

y=sinΘ

Θ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ 0.71	$\frac{\sqrt{3}}{2}$ 0.866	1	0	-1	0

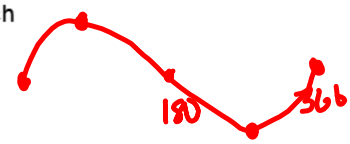




Example 1: Interpreting the Sine Function in Degrees

a. What is the value of y=sinΘ for Θ=270°? -1

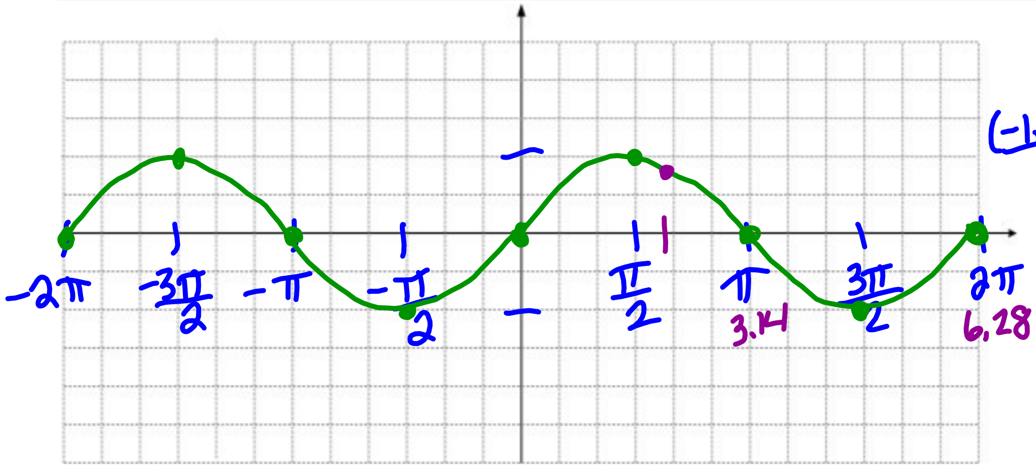
b. For what values of Θ between 0° and 360° does the graph of y=sinΘ reach
   - the maxiumum value of y=1? 90°
   - the minimum value of y=-1? 270°
   - x-intercept of y=0? aka “zero” 0°, 180°, 360°

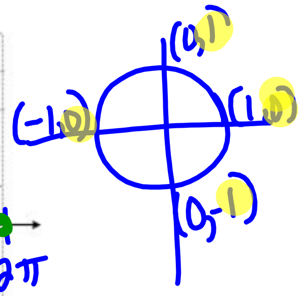


Mathematical convention: An angle measure ϴ can be expressed in degrees or in radians. However, when no unit is mentioned, you should use radians. Let's graph the sine function in radians…

y=sinϴ

		90	180	270	360
ϴ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0





Example 2: Estimating Sine Values in Radians

Use your graph above to estimate the value. Check your estimate with a calculator.

a. sin 2 $\frac{\pi}{3.14}$ 0.8

0.909

b. sin π

0

0

For the sine function, find the following:

a. amplitude

1

b. period (in degrees and radians)

2π

c. domain and range

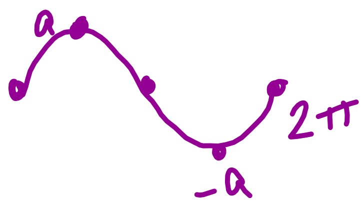
D: ARN

R: $-1 \leq y \leq 1$

$[-1, 1]$

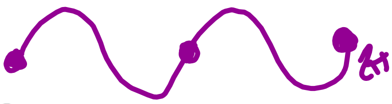
Properties of Sine Functions

Suppose **y = *a* sin *bΘ***, where a≠0, b>0, and Θ in radians.



- The amplitude of the function is $\underline{a}$
- The number of cycles in the interval from 0 to 2π is $\underline{b}$
- The period of the function is $\underline{\frac{2\pi}{b}}$

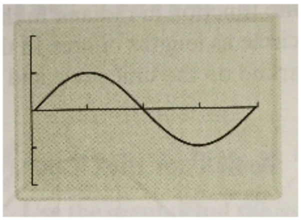
Examples 3&4: Finding the Period and Amplitude of a Sine Function



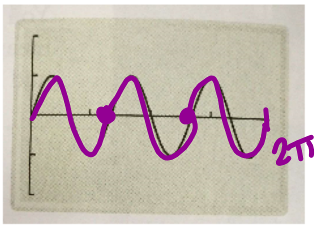
a. Find the amplitude.
b. How many cycles does the sine function have in the interval from 0 to 2π?
c. Find the period.

$b = 2$
$P = \pi$

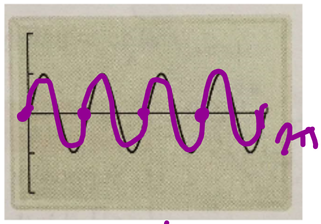
The Θ-axis represents values from 0 to 2π. Each interval on the y-axis represents 1 unit.



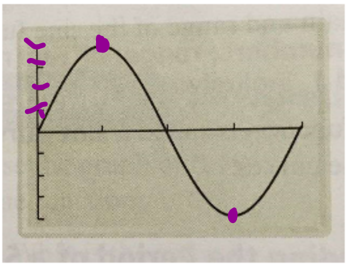
$a = 1$
$b = 1$
$p = 2\pi$



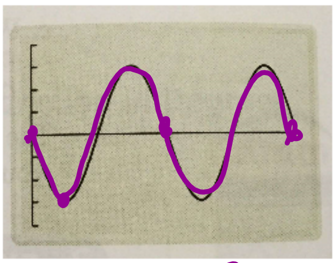
$a = 1$
$b = 3$
$p = \frac{2\pi}{3}$



$a = 1$
$b = 4$
$p = \frac{2\pi}{4} = \frac{\pi}{2}$



$a = 4$
$b = 1$
$p = 2\pi$



$a = -3$
$b = 2$
$p = \frac{2\pi}{2} = \pi$

$$\boxed{\begin{gathered} p = \frac{2\pi}{b} \\ b = \frac{2\pi}{p} \end{gathered}}$$

13.4 The Sine Function (Day 2)

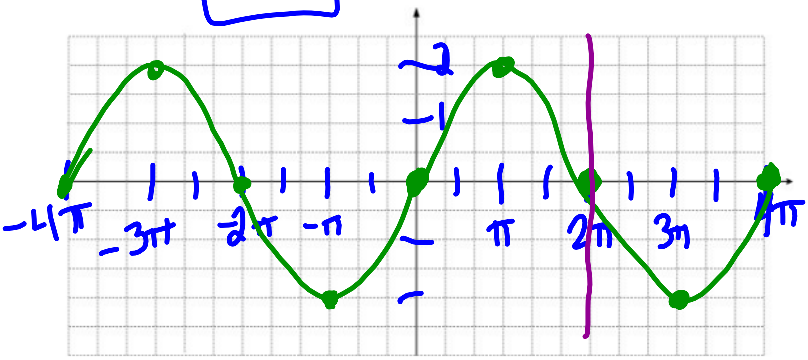
II. Graphing Sine Functions

You can use 5 points equally spaced through one cycle to sketch a sine curve. For a>0, this 5-point pattern is zero-max-zero-min-zero.

Example 5: Sketching a Graph

Sketch one cycle of each sine curve. Then write an equation for each graph.

a. amplitude $\boxed{2, \text{ period } 4\pi}$, a>0



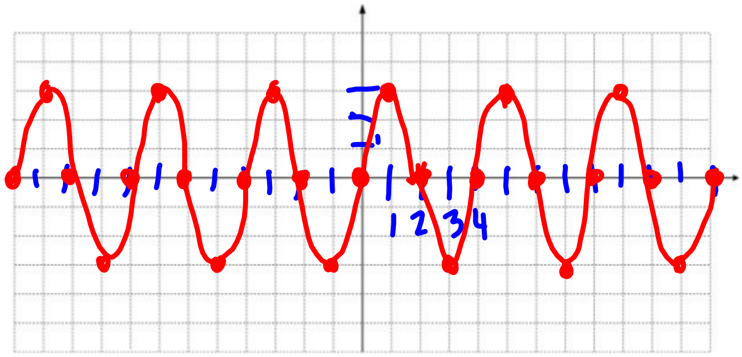
$y = a \sin b\theta$

$y = 2 \sin \frac{1}{2}\theta$

$b = \frac{2\pi}{P}$

$b = \frac{2\pi}{4\pi} = \frac{1}{2}$

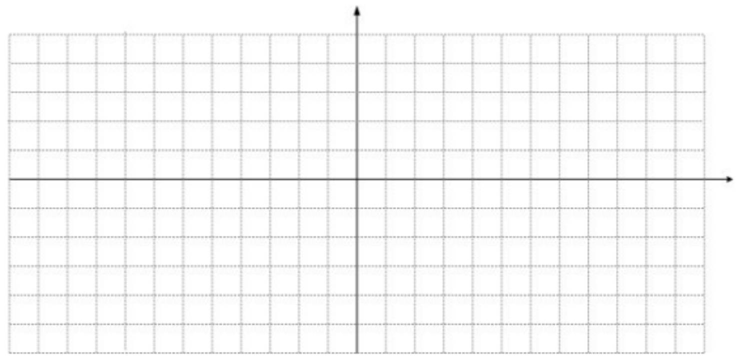
b. amplitude 3, period 4, a>0



$y = 3 \sin \frac{\pi}{2}\theta$

$b = \frac{2\pi}{P} = \frac{2\pi}{4} = \frac{\pi}{2}$

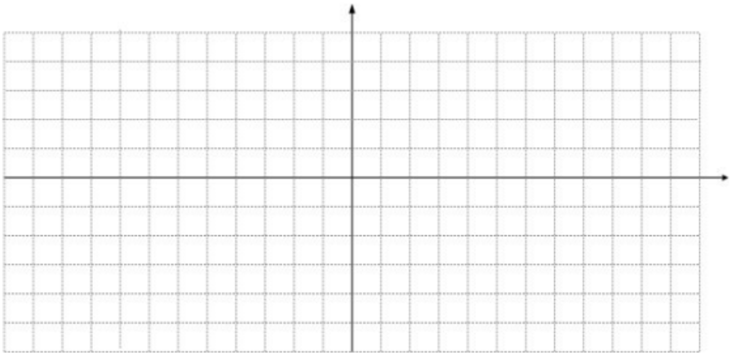
c. Predict the 5-point pattern for the sine function when a<0. Then sketch amplitude 2, period 2π/3.



Example 6: Graphing from a Function Rule

Sketch one cycle of the following sine functions.

1. $y = \frac{1}{2}sin2\theta$



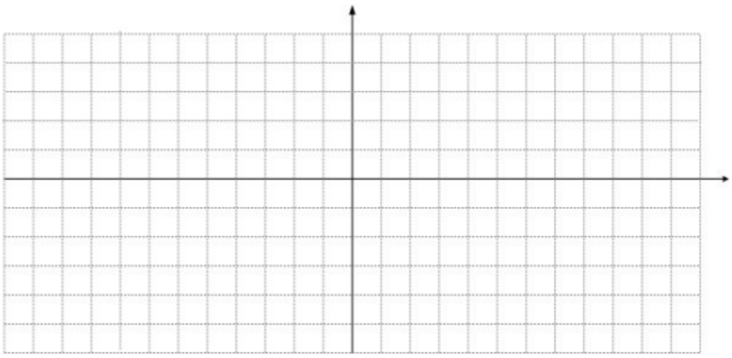
amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:

2. $y = 3sin\frac{\pi}{2}\theta$



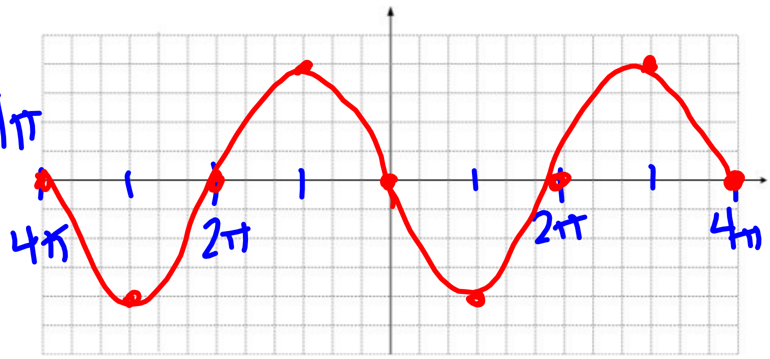
amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:

3. $y = -4sin\frac{1}{2}\theta$



amplitude: $-\boxed{4}$

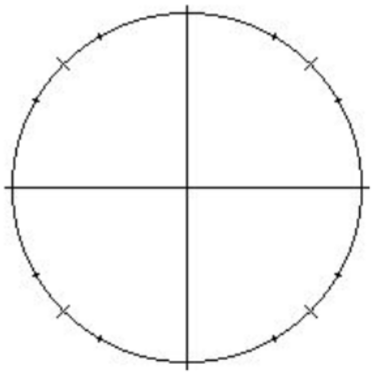
period: $P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

interval spacing on Θ-axis:

5-point pattern:

13.5 The Cosine Function (Day 1)

Warm up: Fill out the unit circle. Evaluate the following angles of cosine.



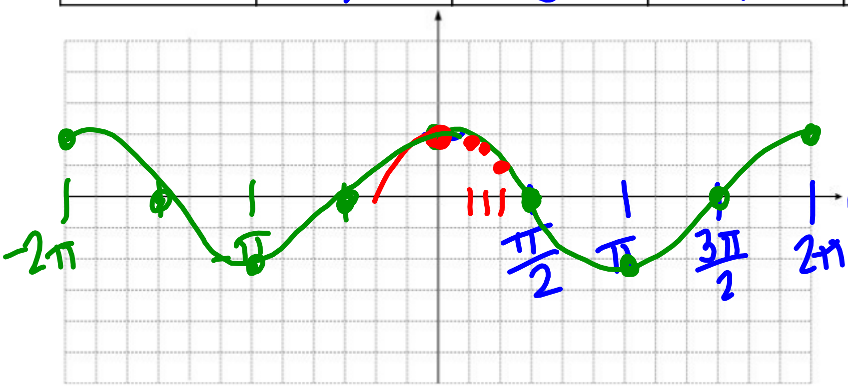
1. $cos\ 0^{\circ}$
2. $cos\ 90^{\circ}$
3. $cos\ 180^{\circ}$
4. $cos\ 270^{\circ}$
5. $cos\ 360^{\circ}$
6. $cos\ 0$
7. $cos\ \frac{\pi}{2}$
8. $cos\ \pi$
9. $cos\ \frac{3\pi}{2}$
10. $cos\ 2\pi$

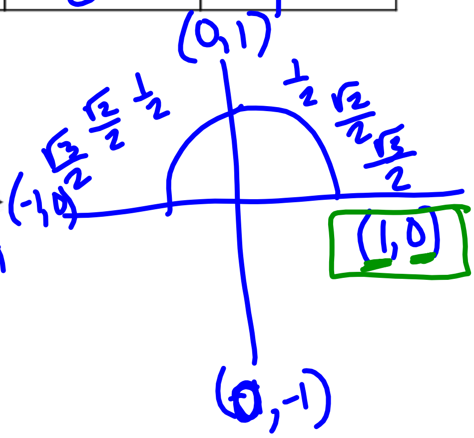
Remember: cosΘ equals the _____-coordinate on the unit circle.

90

I. Graph the cosine function: y=cosΘ

Θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	−1	0	1





Example 1: Interpreting the Graph of y=cosΘ

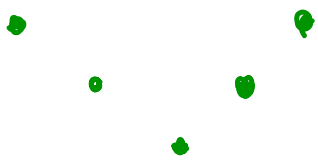
a. Use your graph above. Find the following:

domain period range amplitude

b. In the interval from 0 to 2π, where do the maximum occur? minimum? zeros?

$0, 2\pi$ π $\frac{\pi}{2}, \frac{3\pi}{2}$

c. What is the 5-point pattern of the cosine graph?



Properties of Cosine Functions

Suppose **y = *a* cos *bθ***, where a≠0, b>0, and θ in radians.

- The amplitude of the function is _____
- The number of cycles in the interval from 0 to 2π is _____
- The period of the function is _____

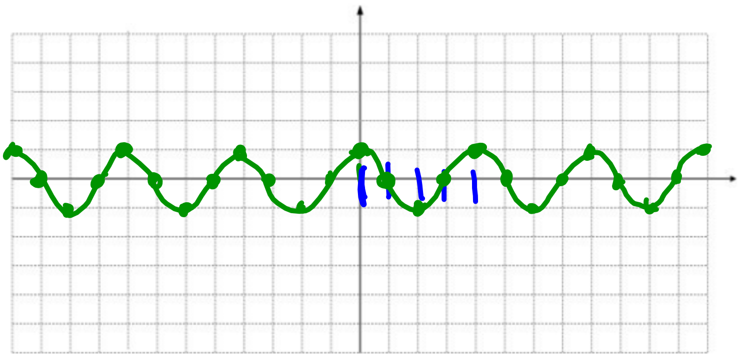
Example 2: Sketching the Graph of a Cosine Function

Sketch one cycle of the following cosine functions.

1. $y = cos\frac{\pi}{2}\theta$

amplitude: 1

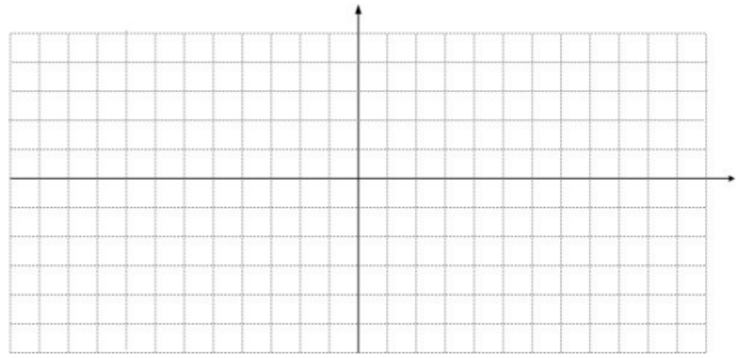
period: $\frac{2\pi}{\frac{\pi}{2}}$ $2\cancel{\pi}\cdot\frac{2}{\cancel{\pi}} = 4$



interval spacing on θ-axis:

5-point pattern:

2. $y = -3cos\theta$



amplitude:

period:

interval spacing on θ-axis:

5-point pattern:

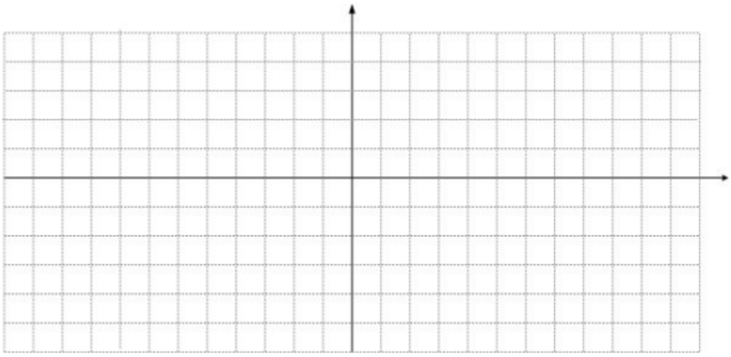
3. $y = 1.5cos2\theta$

amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:



<u>13.5 The Cosine Function (Day 2)</u>

Warm up: Graph the following cosine functions.

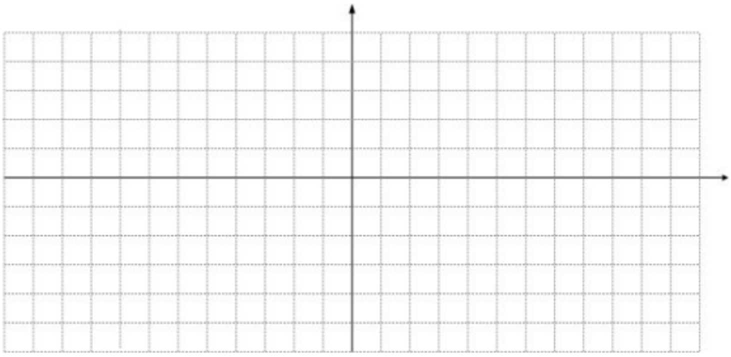
1. $y = 3\cos 2\theta$

amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:



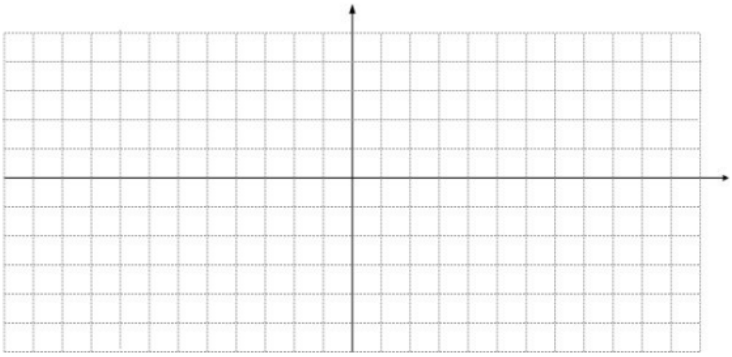
2. $y = -2\cos\theta$

amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:



II. Solving Trigonometric Equations

Example 4: Solving a Cosine Equation

Solve the cosine equation in the interval from 0 to 2π. Round to the nearest hundredth. Calculators needed.

a. $-2\cos\theta = 1.2$

$\frac{-2\cos\theta}{-2} = \frac{1.2}{-2}$

$\cos^{-1}\ \cos\theta = \cos^{-1} -0.6$

$\cos^{-1}(-0.6) = \boxed{2.21}$

b. $3\cos 2t = -2$

$\frac{3\cos 2t}{3} = \frac{-2}{3}$

$\cos^{-1}\ \cos 2t = \cos^{-1} -\frac{2}{3}$

$\frac{2t}{2} = \frac{\cos^{-1}\left(-\frac{2}{3}\right)}{2}$ 2.3

$t = \boxed{1.15}$

c. $5\cos\frac{\pi}{2}t = 3$

Identify the period, range, and amplitude of each function.

22. $y = 3cos\theta$

24. $y = 2cos\frac{1}{2}t$

26. $y = 3cos\left(-\frac{\theta}{3}\right)$

28. $y = 16cos\frac{3\pi}{2}t$

13.6 The Tangent Function

Warm up: Use a calculator to find the sine and cosine of each Θ. Then calculate the ratio of sinΘ to cosΘ.

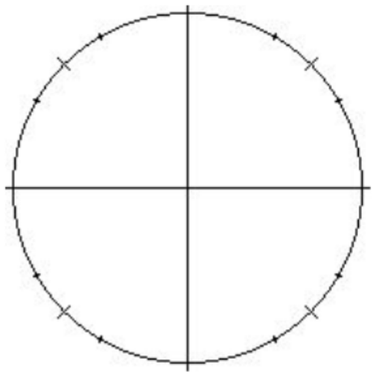
θ	$sin\theta$	$cos\theta$	$\frac{sin\theta}{cos\theta}$
1. $\frac{\pi}{3}$			
2. 30°			
3. 90°			
4. π			
5. $\frac{7\pi}{6}$			

I. The Tangent Function

The cosΘ is derived from the _____ - coordinate of the point on the unit circle.

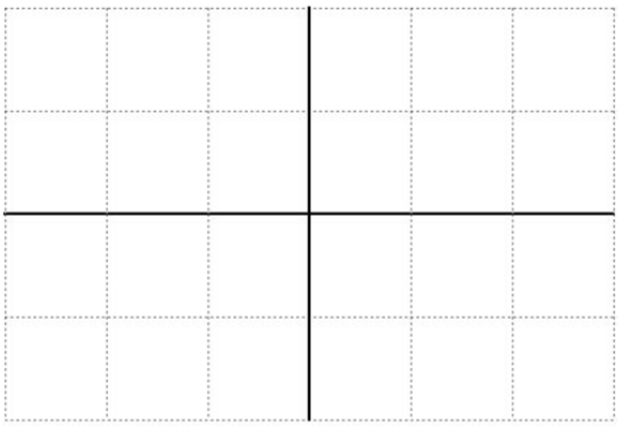
The sinΘ is derived from the _____ - coordinate of the point on the unit circle.

The tanΘ is derived from the ratio of sinΘ to cosΘ. In other words: tanΘ = _________



y=tanΘ

Θ (radians)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$
y							



Features of the parent tangent function:

passes through:

each “branch” is:

x-intercepts (zeros):

vertical asymptotes:

“guide points”:

Properties of Tangent Functions

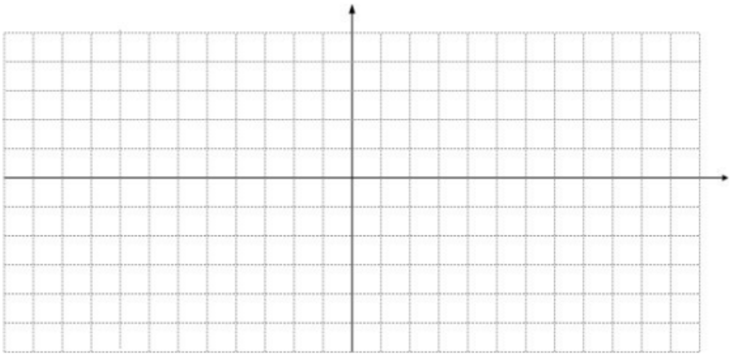
Suppose **y = *a* tan *bΘ***, where b>0, and Θ in radians.

- The period of the function is _____
- 1 cycle occurs in the interval from _________ to _________
- There are vertical aymptotes at each end of the cycle.
- The pattern is *"asymptote, -a, zero, a, asymptote"*.

Example 2: Graphing a Tangent Function

Sketch 2 cycles of each tangent function.

1. $y = tan\pi\theta$



period:

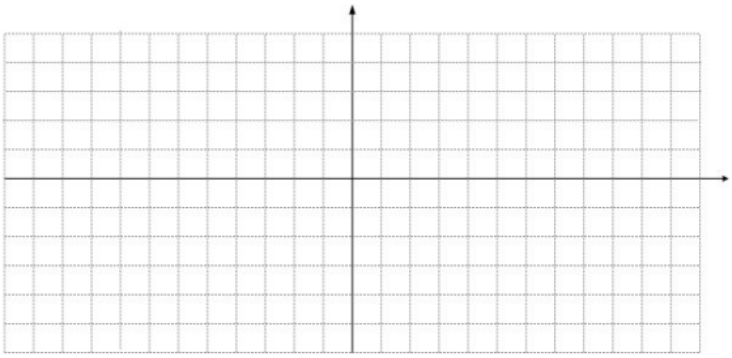
1 cycle: from ______ to ______

VA:

2 guide points:

pattern:

2. $y = tan3\theta$



period:

1 cycle: from ______ to ______

VA:

2 guide points:

pattern:

3. $y = tan\frac{\pi}{2}\theta$

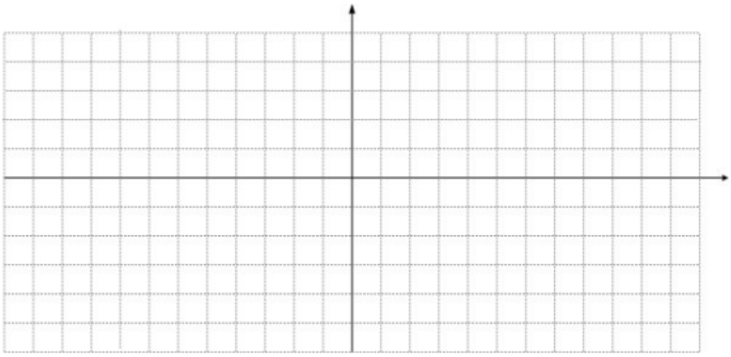
period:

1 cycle: from ______ to ______

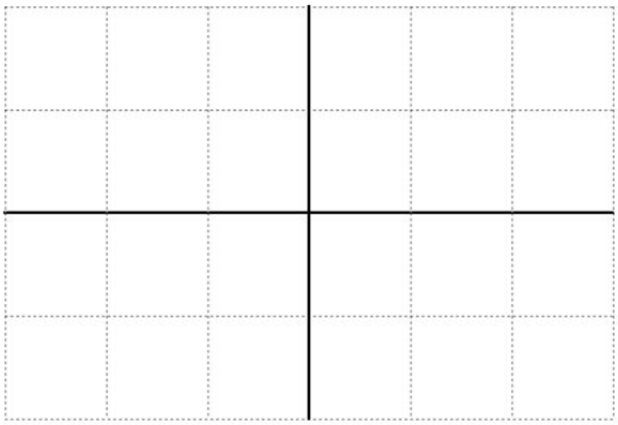
VA:

2 guide points:

pattern:



13.7 Translating Sine and Cosine Functions
Warm Up:
1. Graph y=tanΘ … again! (Try not to peek at prior notes.)



Features of the tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

"guide points":

2. Compare each pair of equations. State the translations (horizontal, vertical) involved.

a. $y = 2x,\ \ y = 2x + 5$ b. $y = |x|,\ \ y = |x + 3|$ c. $y = x^2,\ \ y = x^2 - 4$

d. $y = |x - 2| + 1$ e. $y = x^2,\ \ y = (x + 3)^2 - 6$ d. $y = f(x),\ \ y = f(x - h) + k$

I. Graphing Translations of Trigonometric Functions
Phase shift - the horizontal translation of a function. If *f(x)* is the "parent", then *f(x-h)* translates horizontally *h* units. For example: *f(x-1)* translates ________________, *f(x+3)* translates ________________.
Vertical shift - the vertical translation of a function. If *f(x)* is the "parent", then *f(x)+k* translates vertically k units. For example: *f(x) - 1* translates ________________, *f(x) + 3* translates ________________.

Example 1: Identifying Phase Shifts and Vertical Shifts
What is the value of *h* and *k* in each translation? Describe the shift i.e. "3 units to the left".

a. f(x-2) b. y= cos(x+4) c. f(t-5) d. y= sin(x+3)

e. f(x) - 2 b. y= cosx + 4 c. f(t) - 5 d. y= sinx + 3

Example 2: Graphing Translations

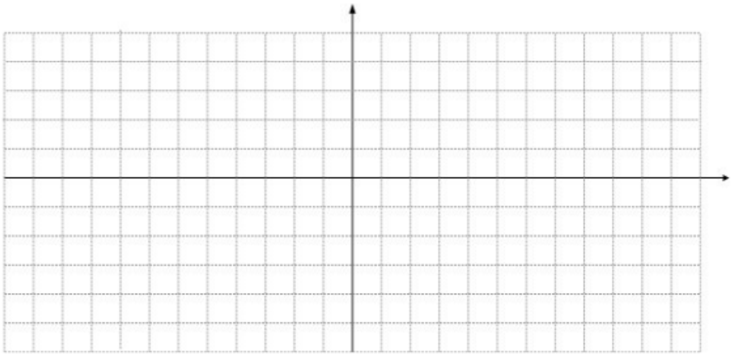
Make a table and then graph the following functions on the same set of axes:

y= sinx

x					
y					

y= sinx + 3

x					
y					

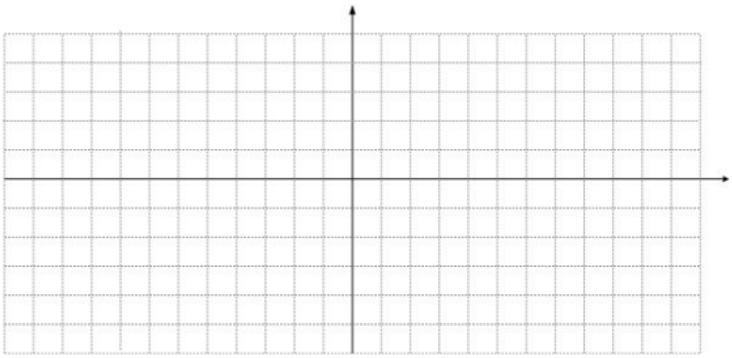


$y = cosx$

x					
y					

$y = cos(x - \frac{\pi}{2})$

x					
y					



Example 3: Graphing a Combined Translation

1. $y = sin(x + \pi) - 2$

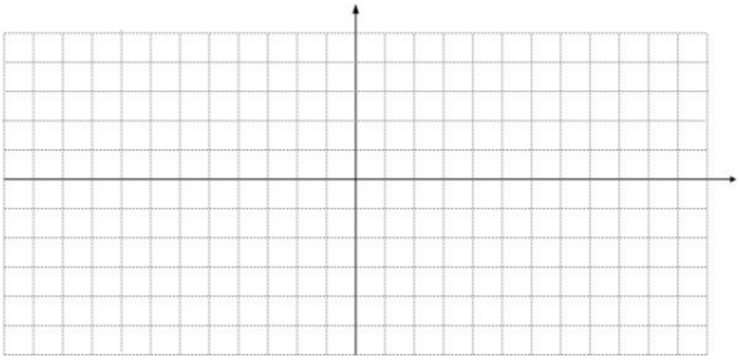
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

2. $y = 2cos(x - \frac{\pi}{2}) + 3$

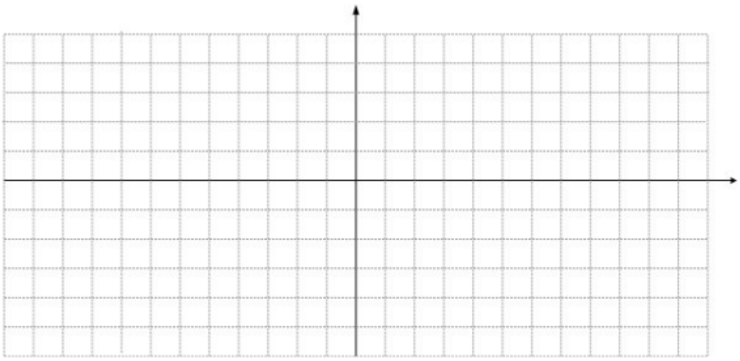
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

<u>Summary: Families of Sine and Cosine Functions</u>

Parent	**Transformed Function**
y=sinx	____________________
y=cosx	____________________

amplitude = h =

period = k =

13.7 Translating Sine and Cosine Functions (Day 2)
Warm Up:

$y = -3sin(x + \frac{\pi}{2}) + 2$

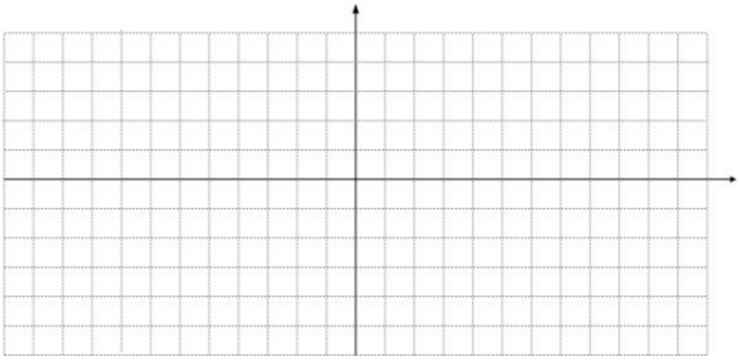
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

When the phase shift is a pesky number...

$y = sin(x - \frac{\pi}{3})$

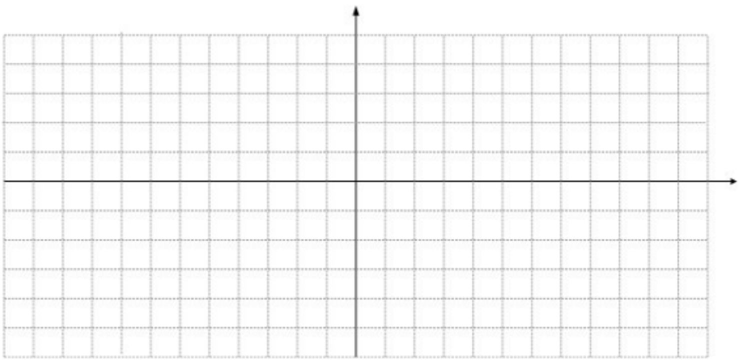
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:

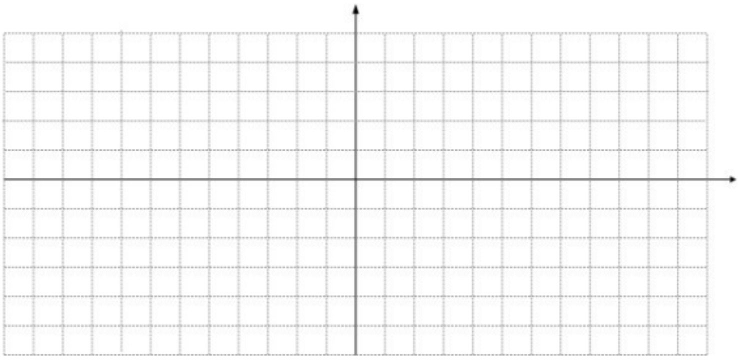


5 point pattern:

Graphing a translation of y = sin 2x...

$y = sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

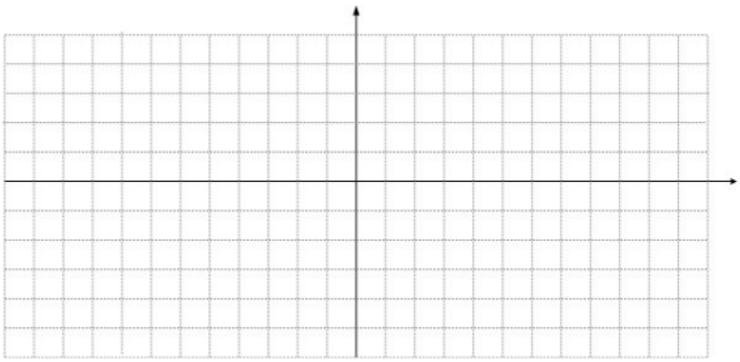
interval spacing on x-axis:

vertical shift:

5 point pattern:

$y = -3sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:

$y = 2cos\frac{\pi}{2}(x + 1) - 3$

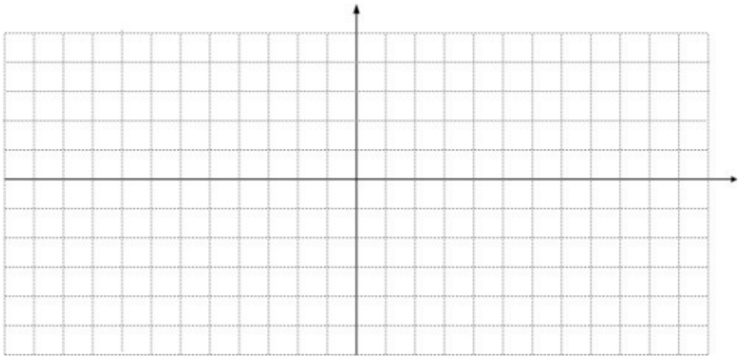
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

Example 5: Writing a Translation

Write an equation for each translation.

a. $y = sin\pi,\ \ \pi\ units\ down$

b. $y =- cosx,\ \ 2\ units\ left$

c. $y = cosx,\ \ \frac{\pi}{2}units\ up$

d. $y = 2sinx,\ \ \frac{\pi}{4}units\ right$

13.8 Reciprocal Trigonometric Functions

Warm up:

Find the reciprocal of each fraction:

1. $\frac{9}{13}$ 2. $-\frac{5}{8}$ 3. $\frac{1}{2\pi}$ 4. $\frac{14}{-t}$ 5. θ

Name the 3 trigonometric functions you have studied so far:

1. ___________
2. ___________
3. ___________

These 3 trigonometric functions have reciprocals.

Definition: Cosecant, Secant, and Cotangent

Example 1: Using Reciprocals

a. Use your calculator (degree mode). Round your answer to the nearest hundredth.

csc 60° cot 55° sec 15°

b. Suppose $cos\theta = \frac{5}{13}$. Find $sec\theta$.

c. Suppose $sin\theta = \frac{-12}{13}$. Find $csc\theta$.

Example 2: Find The Exact Value

csc 30° csc 45° csc 60° csc 90°

sec 30° sec 45° sec 60° sec 90°

cot 30° cot 45° cot 60° cot 90°

Example 3: Using Radians

a. Use your calculator (radian mode). Round your answer to the nearest hundredth.

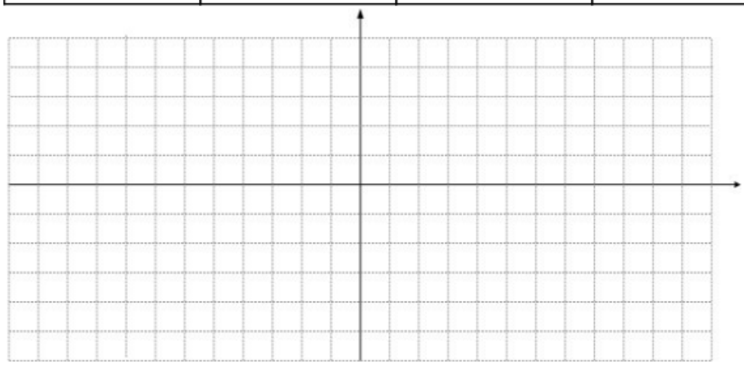
$sec(-1)$ $csc(-1.5)$ $sec2$

b. Find the exact value.

$cot\frac{\pi}{3}$ $cot\pi$ $sec0$

Example 4: Graph The Reciprocal Trigonometric Functions

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= sin x					
y= csc x					



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= cos x					
y= sec x