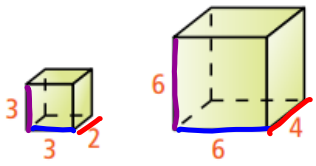
11.7 Area & Volume of Similar Solids

Ex1 | Identifying Similar Solids

Are the two rectangular prisms similar? If so, what is the scale factor of the first figure to the second figure?

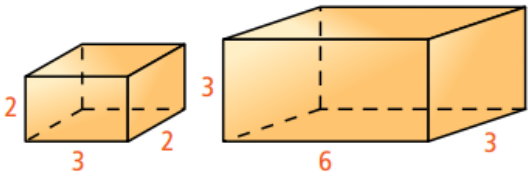
A



$\frac{3}{6} = \frac{3}{6} = \frac{2}{4} = \frac{1}{2}$

1 : 2

B



$\frac{3}{6} \neq \frac{2}{3} = \frac{2}{3}$

not similar

The two similar prisms shown here suggest two important relationships for similar solids.

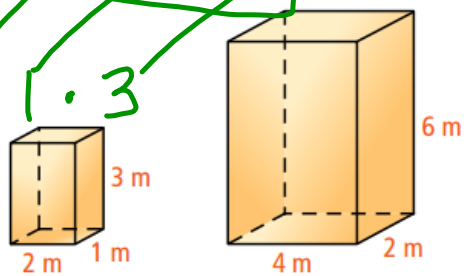
The ratio of the side lengths is 1 : 2

The ratio of the surface areas is 1 : 4 (2^2)

The ratio of the volumes is 1 : 8 (2^3)

2^3 ×2 ×2 ×2

1 · 2 · 3 → 2 · 4 · 6



S.A. = 22 m²
V = 6 m³

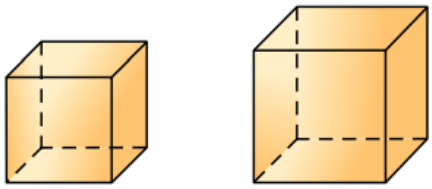
S.A. = 88 m²
V = 48 m³

22 : 88

6 : 48

Ex 2

The square prisms at the right are similar. What is the scale factor of the smaller prism to the larger prism?



1 dim — S.F. → side lengths — $\frac{a}{b}$

2 dim — surface area — $\left(\frac{a}{b}\right)^2$

3 dim — Volume — $\left(\frac{a}{b}\right)^3$

$\frac{729\ cm^3}{1331\ cm^3} = \frac{a^3}{b^3}$

729 [2nd] [$\sqrt[x]{y}$ / y^x] = 9

$\frac{9}{11} = \frac{a}{b}$

Ex 3

Painting The lateral areas of two similar paint cans are 1019 cm^2 and 425 cm^2. The volume of the smaller can is 1157 cm^3. What is the volume of the larger can?

$\frac{1019}{425} = \frac{a^2}{b^2}$

$\frac{\sqrt{1019}}{\sqrt{425}} = \frac{a}{b}$

$\frac{V_{large}}{1157} = \frac{a^3}{b^3}$

$1157 \cdot \frac{V_{large}}{1157} = \left(\frac{\sqrt{1019}}{\sqrt{425}}\right)^3 \cdot 1157$

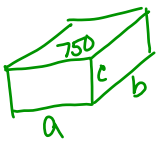
$V_{large} = \left[\left(\frac{1019}{425}\right)\right]^3$

$= (2.4)^3 = (1.55)^3 = 3.7$

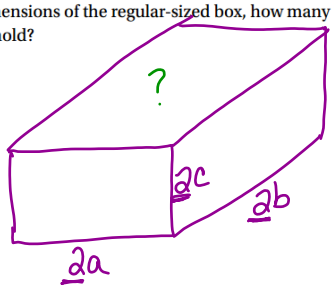
$= 4295.4754$

$4295\ cm^3$

Ex 4 **Packaging** There are 750 toothpicks in a regular-sized box. If a jumbo box is made by doubling all the dimensions of the regular-sized box, how many toothpicks will the jumbo box hold?



1 : 8



$\frac{a \cdot b \cdot c}{2a \cdot 2b \cdot 2c} = \frac{1}{8}$

$\frac{1}{8} = \frac{750}{x}$

$x = 8 \cdot 750$

$x = 6000$ toothpicks

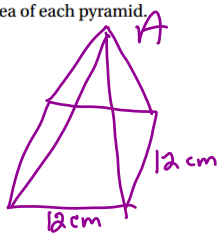
Ex 5 Square pyramids A and B are similar. In pyramid A, each base edge is 12 cm. In pyramid B, each base edge is 3 cm and the volume is 6 cm^3.

a. Find the volume of pyramid A.

b. Find the ratio of the surface area of A to the surface area of B.

c. Find the surface area of each pyramid.

$V = 6\ cm^3$



SF

$\frac{3}{12} = \frac{3}{12} = \frac{1}{4}$

$\frac{a}{b} = \frac{1}{4}$

$\frac{a^3}{b^3} = \frac{1}{64}$

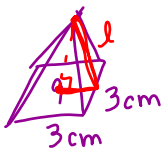
$\frac{1}{64} = \frac{6}{V_A}$

$V_A = 64 \cdot 6$

a. $384\ cm^3$

$\frac{a}{b} = \frac{1}{4} \rightarrow \frac{a^2}{b^2} = \frac{1}{16}$ (b.)

S.A.



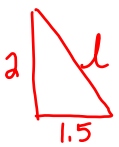
$V = 6\ cm^3$

$V_p = \frac{1}{3}Bh$

$6 = \frac{1}{3} \cdot 9h$

$6 = 3h$

$2 = h$



$2^2 + 1.5^2 = l^2$

$4 + 2.25 = l^2$

$6.25 = l^2$

$2.5 = l$

$SA = B + LA$

$= 9 + 4\left(\frac{1}{2}\right)(3)(2.5)$

$= 9 + 15$

$SA_B = 24\ cm^2$

$SA_A = 384\ cm^2$

$\frac{24}{x} = \frac{1}{16}$

$x = 24 \cdot 16$