

# TRAIN - FASF - 3

## STANDARDIZE

In a certain school, the heights of the population of girls are normally distributed, with a mean of 63 inches and a standard deviation of 2 inches. If there are 450 girls in the school, determine how many of the girls are shorter than 60 inches. Round the answer to the nearest integer.

Enter your answer in the box.

30

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 63}{2} = -\frac{3}{2} = -1.5$$

$$-1.5 \rightarrow 0.0668 \quad (0.0668)(450) = 30.06$$

## INVERSE — Switch x and y; solve for y

Which function is the inverse of  $f(x) = x^3 - 6$ ?

Which function is the inverse of  $f(x) = \frac{1}{2}x - 4$ ?

A.  $f^{-1}(x) = x^3 + 6$

B.  $f^{-1}(x) = \sqrt[3]{x} + 6$

C.  $f^{-1}(x) = \sqrt[3]{x} - 6$

D.  $f^{-1}(x) = \sqrt[3]{x+6}$

$$y = x^3 - 6$$

$$x = y^3 - 6$$

$$\begin{array}{r} x = y^3 - 6 \\ +6 \quad +6 \\ \hline x + 6 = y^3 \end{array}$$

$$\sqrt[3]{x+6} = y = f^{-1}(x)$$

A.  $f^{-1}(x) = \frac{1}{2}x + 2$

B.  $f^{-1}(x) = \frac{1}{2}x + 4$

C.  $f^{-1}(x) = 2x + 4$

D.  $f^{-1}(x) = 2x + 8$

$$y = \frac{1}{2}x - 4$$

$$x = \frac{1}{2}y - 4$$

$$\begin{array}{r} x = \frac{1}{2}y - 4 \\ +4 \quad +4 \\ \hline x + 4 = \frac{1}{2}y \end{array}$$

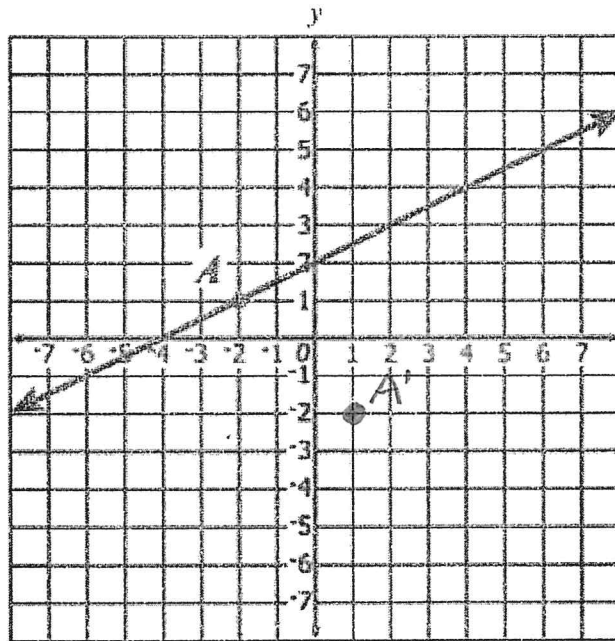
$$2(x+4) = \frac{1}{2}y \cdot 2$$

$$2x+8 = y \rightarrow f^{-1}(x) = 2x+8$$

Point A lies on the graph of  $f(x) = \frac{1}{2}x + 2$ . Locate the image of Point A that lies on the graph  $f^{-1}(x)$ .

A(-2, 1)

A'(1, -2)



The number of *Salmonella* bacteria,  $y$ , in a sample after  $M$  minutes can be found using the equation shown.

$$y = 1,200 \left( 2^{\frac{20}{60}M} \right)$$

To the nearest tenth of a minute, how many minutes will it take for the sample to have 100,000 bacteria?

$$\frac{100000}{1200} = \frac{1200 \left( 2^{\frac{20}{60}M} \right)}{1200}$$

$$83.\bar{3} = 2^{\frac{1}{3}M} \rightarrow 3 \cdot \log_2 83.\bar{3} = \frac{1}{3}M \cdot 3$$

$$M = 3 \log_2 83.\bar{3} \quad (\text{use change of base})$$

$$M = 3 \frac{\log 83.\bar{3}}{\log 2}$$

$$M \approx 19.11$$

# HORIZONTAL ASYMPTOTES

## POLYNOMIALS

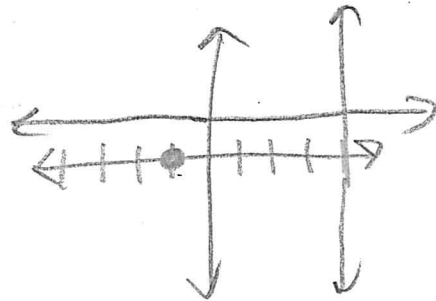
Plug in 1,000 or 1,000,000 (a relatively large number) and estimate the y-value. The horizontal asymptote is equal to this y-value, unless the y-value is a very large (positive or negative) number, in which case, the graph goes to infinity (or negative infinity) and there is no horizontal asymptote.

Generally, for  $f(x) = \frac{ax^m \dots}{bx^n \dots}$ , the horizontal asymptote is

$y = 0$  if  $m < n$ ,

$y = \frac{a}{b}$  if  $m = n$ ,

None, if  $m > n$ ,



A rational function is shown.

$y = \frac{x+1}{x-4}$  same degree, so  $y=1$  H.A.

A. Use the Add Arrow tool to graph the horizontal and/or vertical asymptote(s) of the function.

V.A.: make denominator equal zero

$x-4=0$   
 $x=4$

B. Drag the point to graph the zero(s) of the function.

Zero: If numerator is zero,  $y=0$ , so  $x+1=0 \rightarrow x=-1$

## EXPONENTIAL FUNCTIONS

The horizontal asymptote of the parent is  $y = 0$ , but if and only if the graph is translated up or down, this will change to  $y = k$ , where  $f(x) = a \cdot b^{x-h} + k$  is the exponential function.

What is the equation of the horizontal asymptote of the graph of the following equation?

$f(x) = 4^{(x+1)} - 10$

- A.  $y = 4$
- B.  $y = 0$
- C.  $y = -1$
- D.  $y = -10$

$y = -10$

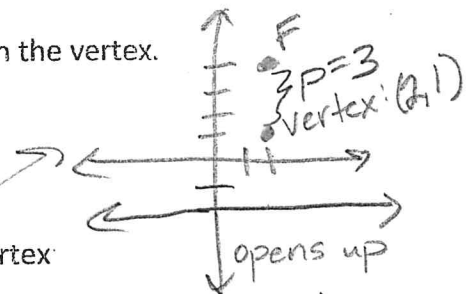
FOCUS and DIRECTRIX: a point and a line which are both equally distant from the vertex.

\*\*\*For  $y = a(x-h)^2 + k \rightarrow$  opens up or down  
 $x = a(y-k)^2 + h \rightarrow$  opens right or left,

$a = \frac{1}{4p}$  where  $p$  is the distance from the focus (or directrix) to the vertex

A parabola has a focus of (2, 4) and a directrix of  $y = -2$ .

What is the equation for the parabola?

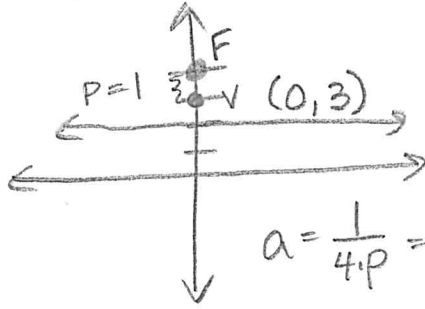


$y = a(x-2)^2 + 1$

$a = \frac{1}{4 \cdot 3} = \frac{1}{12}$   
 $y = \frac{1}{12}(x-2)^2 + 1$

Which equation represents a parabola with a focus of (0, 4) and a directrix of  $y = 2$ ?

- A.  $y = x^2 + 3$
- B.  $y = -x^2 + 1$
- C.  $y = \frac{x^2}{2} + 3$
- D.  $y = \frac{x^2}{4} + 3$



$$y = a(x-h)^2 + k$$

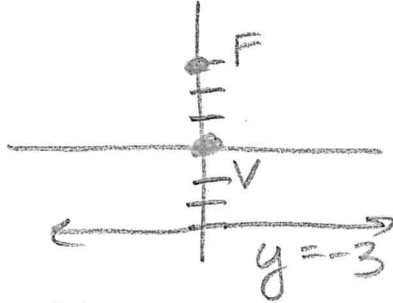
$$y = \frac{1}{4}(x-0)^2 + 3$$

$$y = \frac{1}{4}x^2 + 3$$

$$a = \frac{1}{4p} = \frac{1}{4 \cdot 1} = \frac{1}{4}$$

A parabola has focus at (0, 3) and vertex at the origin. Which could be the equation of the directrix?

- A.  $y = -12$
- B.  $y = -3$
- C.  $y = 0$
- D.  $y = 3$



TRIGONOMETRY

$\arcsin x = \sin^{-1} x =$  the inverse of  $\sin x$   
 $\arccos x = \cos^{-1} x =$  the inverse of  $\cos x$   
 $\arctan x = \tan^{-1} x =$  the inverse of  $\tan x$

The value of  $\sin$ ,  $\cos$ , or  $\tan$  will be given and you need to find the angle using the unit circle or by using the button on your calculator to find the angle value.

Ex: What is  $\arcsin \frac{1}{2}$ ? Since  $\sin 30 = \frac{1}{2}$ ,  $\arcsin \frac{1}{2} = \underline{30}$

Find  $\cos^{-1}.89$  to the nearest degree.  $\rightarrow \arcsin .89$  or  $\cos^{-1}.89$   
 $\approx 27^\circ$   
 In calculator, enter .89 then 2nd  $\rightarrow \cos^{-1}$

Don't forget!

- $\pi$  radians =  $180^\circ$
- The radian measure of a central angle equals the arc length the angle intercepts on the unit circle. If the circle does not have a radius of one, the arc length would be multiplied by  $r$ .  
 $\text{radian measure} \cdot \text{radius} = \text{arc length}$

Which degree measure is equivalent to  $\frac{11\pi}{18}$ ?

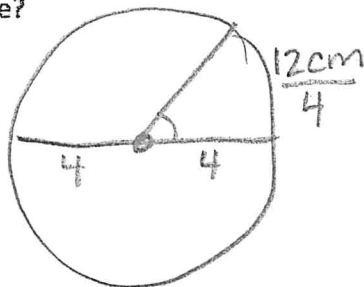
- A.  $220^\circ$
- B.  $110^\circ$
- C.  $55^\circ$
- D.  $10^\circ$

$$\frac{11\pi}{18} \cdot \frac{180^\circ}{\pi} = 110^\circ$$

$\pi$  radians =  $180^\circ$   
 \* Important to remember

The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

- A.  $\frac{3}{2}$
- B. 3
- C. 4
- D.  $8\pi$



arc length =  $r \cdot \text{radian measure}$

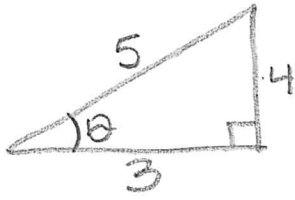
$$\frac{12}{4} = \frac{4 \cdot \text{radian measure}}{4}$$

$\underline{3} = \text{radian measure}$

DRAW RIGHT TRIANGLES and USE the PYTHAGOREAN THEOREM

Angle  $\theta$  is in Quadrant II, and  $\sin\theta = \frac{4}{5}$ . What is the value of  $\cos\theta$ ?

- A.  $\frac{4}{5}$
- B.  $\frac{3}{5}$
- C.  $-\frac{3}{5}$
- D.  $-\frac{4}{5}$



$$\sin\theta = \frac{4}{5} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$a^2 + 4^2 = 5^2$$

$$a^2 + 16 = 25$$

$$a^2 = 9$$

$$a = \pm 3$$

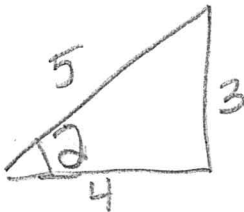
$$= \frac{3}{5}$$

QII  
cos neg  
 $-\frac{3}{5}$



Angle  $\theta$  is in Quadrant IV, with  $\cos\theta = \frac{4}{5}$ . What is  $\sin\theta$ ?

- A.  $\frac{3}{4}$
- B.  $-\frac{3}{5}$
- C.  $\frac{9}{25}$
- D.  $\frac{3}{5}$



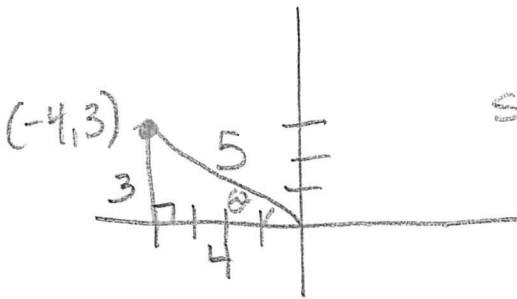
$$\sin\theta = \frac{3}{5} \quad \text{QIV} \rightarrow \sin\theta = -\frac{3}{5}$$

QIV  
sin is neg

S	A
T	C

If the terminal side of angle  $\theta$ , in standard position, passes through point  $(-4, 3)$ , what is the numerical value of  $\sin\theta$ ?

- A.  $\frac{3}{5}$
- B.  $\frac{4}{5}$
- C.  $-\frac{3}{5}$
- D.  $-\frac{4}{5}$



$$\sin\theta = \frac{3}{5} \rightarrow \text{positive}$$

S	A
T	C

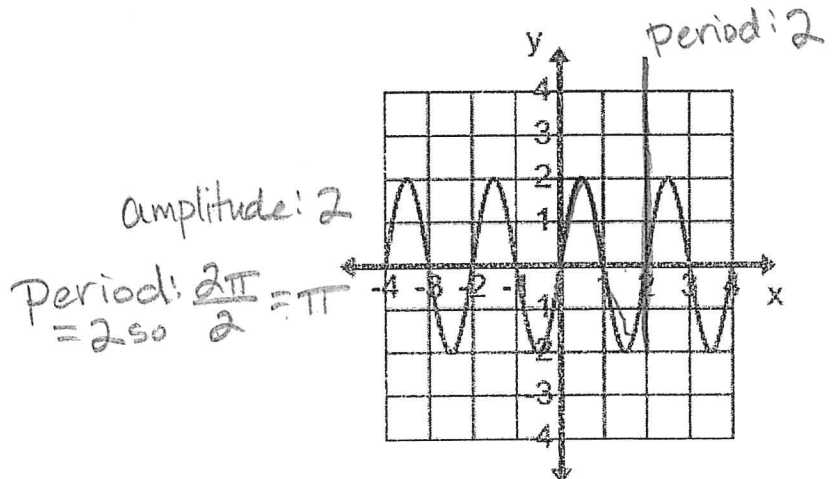
Which statement is incorrect for the graph of the function  $y = -3\cos\left(\frac{\pi}{3}(x-4)\right) + 7$ ?

- A. The period is 6. ✓
- B. The amplitude is 3. ✓
- C. The range is  $[4, 10]$ . ✓
- D. The midline is  $y = -4$ .

period  $\frac{2\pi}{\frac{\pi}{3}} = 6$  up 7 → midline  $y = 7$   
 amplitude: 3 right 4  
 Range:  $7 \pm 3 \rightarrow 10$   
 $\downarrow$   
 4

Which is the equation of the graph shown below?

- A.  $f(x) = 2 \sin \pi x$
- B.  $f(x) = 2 \sin 2\pi x$
- C.  $f(x) = \frac{1}{2} \sin \pi x$
- D.  $f(x) = \frac{1}{2} \sin \frac{\pi}{2} x$

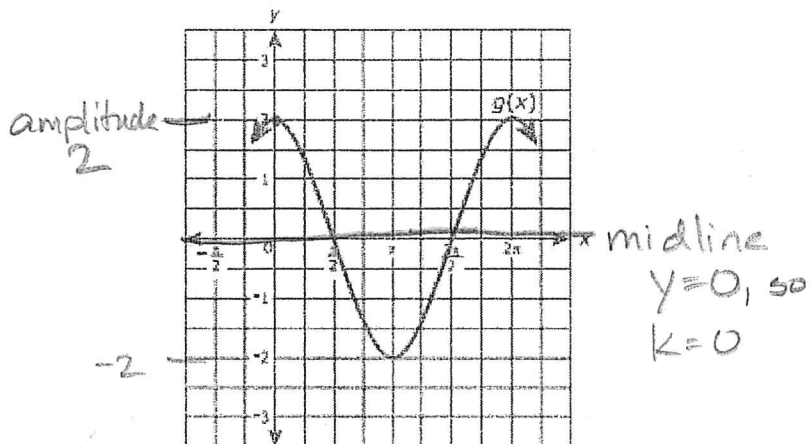


Part A

The function  $f(x) = \cos(x)$ . Function  $g(x)$  results from a transformation on the function  $f(x) = \cos(x)$ . A portion of the graph of  $g(x)$  is shown.

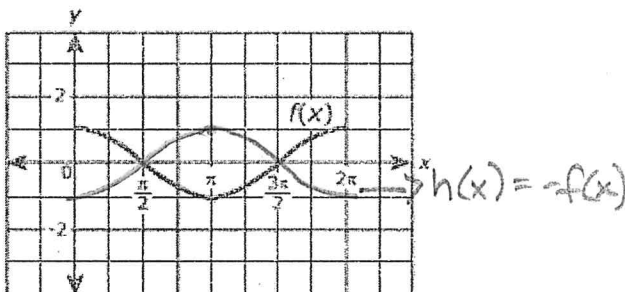
What is the equation of  $g(x)$ ?

- A.  $g(x) = \cos(x) - 2$
- B.  $g(x) = \cos(x) + 2$
- C.  $g(x) = \cos(2x) + 0$
- D.  $g(x) = 2\cos(x) + 0$



Part B

The graph shows  $f(x) = \cos(x)$  on the interval  $0 \leq x \leq 2\pi$ .



Function  $h$  is a transformation of such that  $h(x) = -f(x)$ . Which of the following statements is true? Select EACH correct statement.

- Function  $f$  is an even function.
- Function  $f$  is an odd function.
- Function  $f$  is neither an even nor odd function.
- Function  $h$  is an even function.
- Function  $h$  is an odd function.
- Function  $h$  is neither an even nor odd function.

A wave on an oscilloscope has an amplitude of 2 millimeters and a frequency of 550 cycles per second. The wave can be modeled by a cosine function. Which equation best represents  $h$ , the height in millimeters from the equilibrium position, as a function of  $t$ , the time in seconds?

not A or B  
550 cycles per second  
period:  $550 \cdot 2\pi = 1100\pi$

- A.  $h = \cos(550\pi t)$
- B.  $h = \cos(1100\pi t)$
- C.  $h = 2\cos(550\pi t)$
- D.  $h = 2\cos(1100\pi t)$

A wheel of Matthew's bicycle has a radius of 1 foot. He uses chalk to create a marking on the outer edge of the wheel. Matthew rides his bicycle at a constant speed so that the wheel rotates 3 times every second.

Which function,  $h(t)$ , represents the height, in feet, of the marking from the ground with respect to time,  $t$ , in seconds?

A.  $h(t) = \sin\left(\frac{2\pi}{3}t\right)$

B.  $h(t) = \sin\left(\frac{2\pi}{3}t\right) + 1$

C.  $h(t) = \sin(6\pi t)$

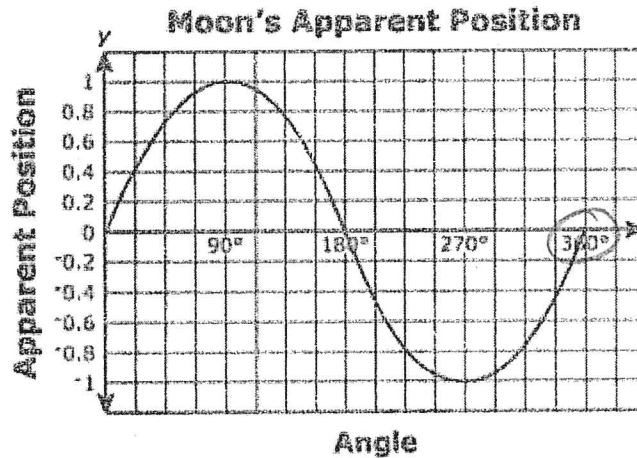
D.  $h(t) = \sin(6\pi t) + 1$



midline  $y=1$   
 $k=1$   
 $y = \sin(6\pi t) + 1$

period:  $\frac{1}{3}$  sec  $\rightarrow \frac{2\pi}{\frac{1}{3}} = 6\pi$

The apparent position of a moon varies sinusoidally with the changing angle from a line of sight as it orbits Jupiter. The moon's apparent position is shown in the graph below.



amplitude = 1  
period:  $360^\circ$

Which are the closest amplitude and period of the moon's orbit?

- A. Amplitude = 0.5 and Period =  $180^\circ$
- B. Amplitude = 0.5 and Period =  $360^\circ$
- C. Amplitude = 1 and Period =  $180^\circ$
- D. Amplitude = 1 and Period =  $360^\circ$

The graph of which function has a period of  $\pi$  and an amplitude of  $\pi$ ?

A.  $y = \frac{1}{\pi} \sin 2x$

B.  $y = \pi \sin 2x$

C.  $y = \frac{1}{\pi} \sin \frac{1}{2}x$

D.  $y = \pi \sin \frac{1}{2}x$

$\frac{2\pi}{\pi} = 2$

USE THE FORMULA SHEET

The probability that Flight 9876 will be late is 0.27. The probability that Flight 123 will be late is 0.11. The probability that both flights will be late is 0.09. What is the probability that Flight 9876 or Flight 123 will be late?

- A. 0.47
- B. 0.38
- C. 0.29
- D. 0.07

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$0.27 + 0.11 - 0.09 =$$

$$0.38 - 0.09 = 0.29$$

WHEN VALUES FOR X and Y ARE GIVEN, PLUG THEM IN TO FIND AN EQUATION.

Paul started to train for a marathon. The table shows the number of miles Paul ran during each of the first three weeks after he began training.

Week	1	2	3
Distance (miles)	10	12	14.4

If this pattern continues, which of the listed statements could model the number of miles Paul runs  $a_n$ , in terms of the number of weeks,  $n$ , after he began training? Select ALL that apply.

- $a_n = 10 + 2(n - 1)$
- $a_n = 10n^2$
- $a_n = 10(1.2)^{n-1}$
- $a_1 = 10, \quad a_n = 1.2a_{n-1}$
- $a_1 = 10, \quad a_n = 2 + a_{n-1}$

Plug in 1 → get 10?

$$a_n = 10 + 2(0) = 10 \checkmark$$

$$a_n = 10(1)^2 = 10 \checkmark$$

$$a_n = 10(1.2)^{1-1} = 10 \cdot 1 = 10 \checkmark$$

$$a_n = 10 \checkmark$$

$$a_1 = 10 \checkmark$$

Plug in 2 → get 12?

$$a_2 = 10 + 2(2-1) = 10 + 2 = 12 \checkmark$$

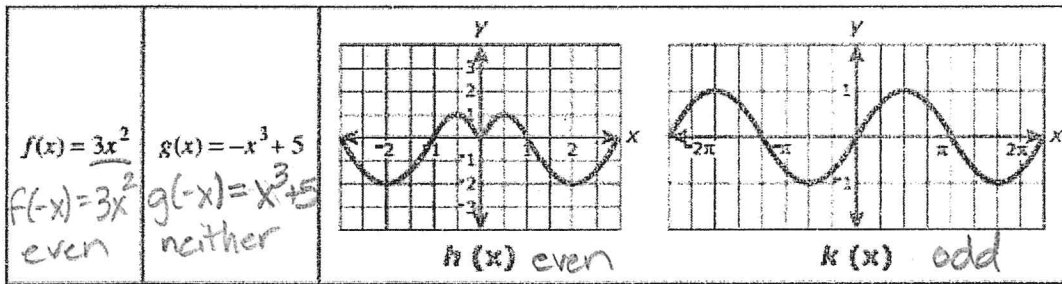
$$a_2 = 10 \cdot 2^2 = 40 \times$$

$$a_2 = 10(1.2)^{2-1} = 10(1.2)^1 = 12$$

$$a_2 = 1.2 \cdot 10 = 12$$

$$a_2 = 2 + 10 = 12$$

Consider the functions  $f(x)$  and  $g(x)$  described by the equations and the functions  $h(x)$  and  $k(x)$  shown by graphs.



$$a_3 = 10 + 2(2) = 14 \times$$

$$a_3 = 10(1.2)^{3-1} = 10(1.2)^2 = 14.4 \checkmark$$

$$a_3 = 10(1.2)^{3-1} = 10(1.2)^2 = 10(1.44) = 14.4 \checkmark$$

$$a_3 = 2 + 12 = 14 \times$$

Which of the statements are true? Select all that apply.

- $f$  is an odd function.
- $f$  is neither an even nor odd function.
- $g$  is an even function.
- $g$  is neither an even nor odd function.
- $h$  is an even function.
- $h$  is an odd function.
- $k$  is an odd function.

Which equation has the same solution as  $\log_4(x+7) = 5$ ?

- A.  $4^{x+7} = 5$
- B.  $5^{x+7} = 5$
- C.  $5^4 = x+7$
- D.  $5^4 = x+7$

none

$$4^5 = x+7$$

Aaron invested \$4000 in an account that paid an interest rate  $r$  compounded continuously. After 10 years he has \$5809.81. The compound interest formula is  $A = Pe^{rt}$ , where  $P$  is the principal (the initial investment),  $A$  is the total amount of money (principal plus interest),  $r$  is the annual interest rate, and  $t$  is the time in years.

Part A

Divide both sides of the formula by  $P$  and then use logarithms to rewrite the formula without an exponent. Show your work.

$$\frac{5809.81}{4000} = \frac{4000e^{rt}}{4000}$$

$$1.4524525 = e^{10r}$$

Part B

Using your answer for Part A as a starting point, solve the compound interest formula for the interest rate  $r$ .

$$\ln \frac{1.4524525}{1} = \frac{10r}{1}$$

$$r = \frac{\ln 1.4524525}{10}$$

Part C

Use your equation from Part A to determine the interest rate.

$$r = 3.73\%$$

$$r = 0.037325$$

Which table(s) represent a function with the same y-intercept as  $f(x) = 2^x$ ?

y-int means  $x=0$

x	y
1	4
2	8
3	16
4	32
5	64

x	y
1	4
2	16
3	64
4	256
5	1,024

x	y
1	1
2	2
3	4
4	8
5	16

$$f(x) = 2^0 = 1$$

$0 \leq x$   
 $\cdot 2$   
 $\cdot 2$   
 $\cdot 2$   
 $\cdot 2$

- A. table 2 only
- B. table 3 only
- C. tables 1 and 2
- D. tables 1 and 3

The mean population of the counties in Florida is 289,294 and the standard deviation is 461,176. It can be assumed that the population is approximately normally distributed.

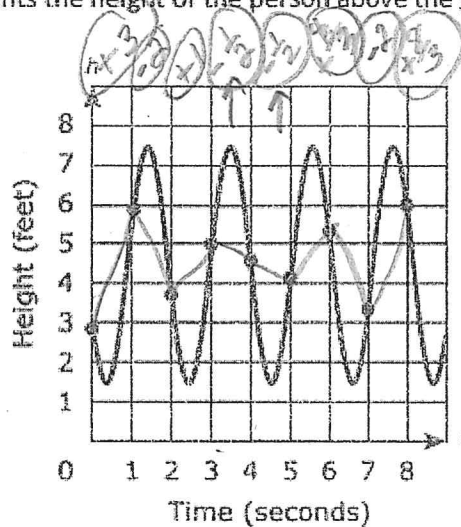
What percentage of the population of all counties is within one standard deviation of the mean?

68%

always 68%



The graph models the height  $h$  above the ground, in feet, at time  $t$ , in seconds, of a person swinging on a swing. Each point indicated on the graph represents the height of the person above the ground at the end of each one-second interval.



Select two time intervals for which the average rate of change in the height of the person is approximately  $-\frac{1}{2}$  feet per second.

- from 0 seconds to 1 second
- from 1 second to 2 seconds
- from 2 seconds to 3 seconds
- from 3 seconds to 4 seconds
- from 4 seconds to 5 seconds
- from 5 seconds to 6 seconds
- from 6 seconds to 7 seconds

Select each statement that is true about the graph of  $f(x) = \sin(x + 3) - 2$ .  $\rightarrow$  *midline  $y = -2$*

- Amplitude: 1  $\checkmark$  *amplitude: 1*
- Amplitude: 2  $\times$
- Midline:  $y = 2$   $\times$
- $y$ -intercept:  $(0, -2)$   $\times$   *$\sin(0+3) - 2 \stackrel{?}{=} -2$  ;  $\sin(0+3) - 2 \stackrel{?}{=} 0$*
- $x$ -intercept:  $(0, 0)$   $\times$   *$\sin 3 - 2 \neq -2$  ;  $\sin 3 - 2 \neq 0$*

A quadratic equation is shown.

$$-3x^2 + 5x + 4 = 4x^2 + 2x + 13$$

$$\begin{array}{r} +3x^2 - 5x - 4 \\ +3x^2 - 5x - 4 \\ \hline \end{array}$$

Drag values to the boxes to complete the solution to the equation.

$$\begin{aligned} 0 &= 7x^2 - 3x + 9 \\ x &= \frac{3 \pm \sqrt{9 - 4 \cdot 7 \cdot 9}}{2 \cdot 7} \\ &= \frac{3 \pm \sqrt{9 - 252}}{14} = \frac{3 \pm \sqrt{-243}}{14} = \frac{3 \pm 9i\sqrt{3}}{14} \end{aligned}$$

*Handwritten calculations:  $63$ ,  $1.63$ ,  $3.21$ ,  $7.9$ ,  $3.81$ ,  $9$ .*

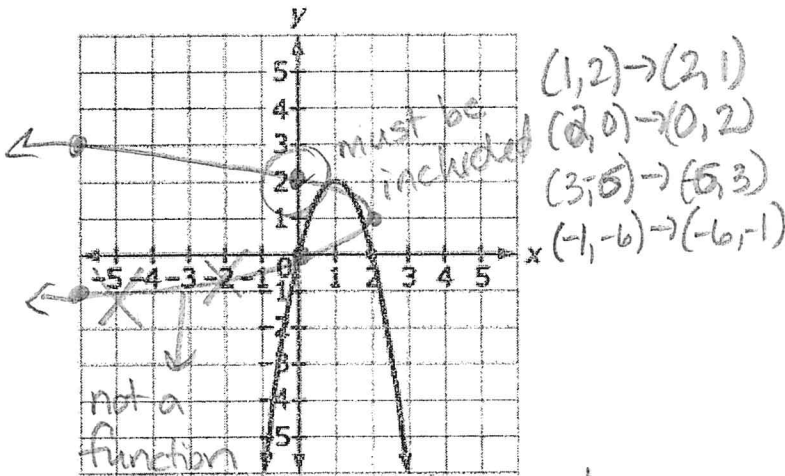
A coffee machine makes one cup of coffee at a time. The amount of coffee the machine makes can be selected before the coffee is made. Jerry selects the coffee machine's 14.5-ounce option for his 16-ounce cup. The amount of coffee, in ounces, in his cup at time  $x$  is given by the function  $C(x) = -0.4x^2 + 5.06x$ , where  $x$  is in minutes.

What is the largest domain for which  $C(x)$  models the amount of coffee in a cup?  $[0, 12.65] \rightarrow$  *seconds*

$$\begin{aligned} -0.4x(x - 12.65) &= 0 \\ -0.4x &= 0 & x - 12.65 &= 0 \\ x &= 0 & x &= 12.65 \end{aligned}$$

A function  $f(x)$  that contains points A, B, C, and D is shown.

A quadratic function  $f(x)$  is shown.



$(1, 2) \rightarrow (2, 1)$   
 $(0, 0) \rightarrow (0, 2)$   
 $(3, 5) \rightarrow (5, 3)$   
 $(-1, -6) \rightarrow (-6, -1)$

with both parts and

Select symbols and values to restrict the domain of  $f(x)$  so that  $f^{-1}(x)$ , the inverse of  $f(x)$ , is a function and the domain of  $f^{-1}(x)$  includes  $x = 2$ .  $(-\infty, 2]$

An equation is shown.  $7^{\frac{19}{4}} \cdot 7^{\frac{b}{a}} = 7^{\frac{9}{4}} \cdot 7^{\frac{3}{2}}$   
 $7^{\frac{19}{4}} \cdot \sqrt[4]{7^b} = 7^{\frac{9}{4}} \cdot \sqrt{7^3}$   $7^{\frac{19}{4}} + \frac{b}{a} = 7^{\frac{9}{4}} + \frac{3}{2} \cdot \frac{2}{2} \rightarrow \frac{6}{4}$

What are possible values for  $a$  and  $b$  that make this equation true?

$a = -1, b = 1$  or  $\frac{19}{4} + \frac{b}{a} = \frac{15}{4} - \frac{19}{4}$   
 $a = 1, b = -1$   $\frac{b}{a} = -\frac{4}{4} = -1$

$\frac{3x^3 + 5x}{x+1} = Ax^2 + Bx + C + \frac{R(x)}{Q(x)}$

What are the values of  $B$ ,  $R(x)$ , and  $Q(x)$  that make the equation true?

$B = -3, R(x) = -8, Q(x) = x + 1$

Marcy plays a game with a spinner and a 6-sided number cube. The spinner is divided into 4 equal sections numbered 1 to 4. Marcy spins the spinner and rolls the number cube. She records both numbers.

What is the probability that at least one of the numbers is a 4?  $P(4_c \text{ or } 4_s) = P(4_c) + P(4_s)$  independent  $\frac{1}{6} + \frac{1}{4} = \frac{5}{12}$   
 $\frac{2}{12} + \frac{2}{12}$

Jared is opening several ice cream stores. The walls of the stores can be yellow or blue. He designs an experimental study to determine if the color of the walls affects how much ice cream people eat.

Jared finds 164 volunteers. He randomly assigns half of them to a room with yellow walls and lets them eat as much chocolate ice cream as they want for one hour. He assigns the other half to a room with blue walls and lets them eat as much vanilla ice cream as they want for one hour.

Jared records the total amount of ice cream eaten in each room.

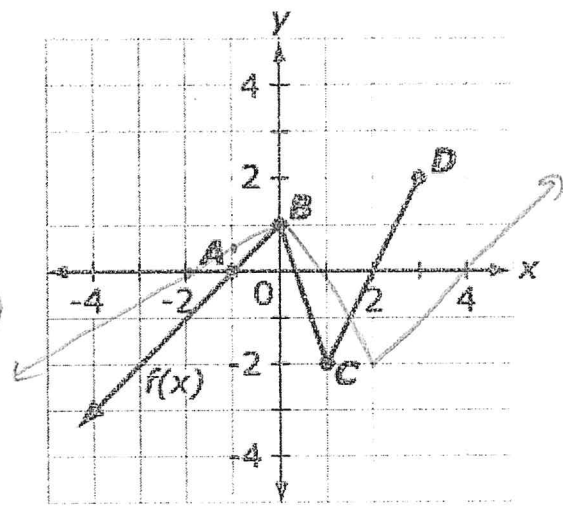
What is one flaw in Jared's study?

There are too many volunteers.

The room assignments were random.

The groups had different flavors of ice cream.

The groups did not have different numbers of volunteers.



The function is transformed to create the function  $g(x)$  such that  $g(x) = f(\frac{1}{2}x) - 5$ .  $\rightarrow$  down 5 units  
 horizontal stretch  $\rightarrow 2$

Complete the table to show the coordinates of points  $A'$ ,  $B'$ ,  $C'$ , and  $D'$ .

$A(-1, 0)$ $\times 2 = -5$	$A'(-2, -5)$
$B(0, 1)$ $\times 2 = -5$	$B'(0, -4)$
$C(1, -2)$ $\times 2 = -5$	$C'(2, -7)$
$D(3, 2)$ $\times 2 = -5$	$D'(6, -3)$

$$\begin{array}{r} -11 \quad 3 \quad 2 \quad 1 \quad 0 \\ 3 \quad 0 \quad 5 \quad 0 \\ -3 \quad 3 \quad -8 \\ \hline 3 \quad -3 \quad 8 \quad -8 \\ 3x^2 - 3x + 8 + \frac{-8}{x+1} \end{array}$$

The table shows information about 10 students in Mrs. McKeon's calculus class.

Name	Gender	In Math Club?
Alexa	Female	Yes
Carlos	Male	Yes
Keisha	Female	Yes
Kumiko	Female	No
Lisette	Female	No
Michael	Male	Yes
Paolo	Male	No
Radha	Female	Yes
Thomas	Male	No
Xavier	Male	Yes

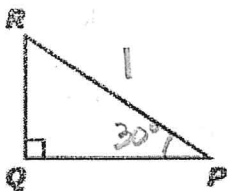
Mrs. McKeon randomly picks two students to present a homework problem. She defines two events as shown.

- Event  $E$ : A male and a female are selected.
- Event  $F$ : Both of the students are in the math club.

Select all the sets of students that are in the complement of the union of events  $E$  and  $F$ .  $\rightarrow$  means not in either

	<u>Event E</u>	<u>Event F</u>
Lisette and Thomas $\rightarrow$	✓	X
Keisha and Kumiko $\rightarrow$	X	X
Radha and Thomas $\rightarrow$	✓	X
Carlos and Michael $\rightarrow$	X	✓
Kumiko and Lisette $\rightarrow$	X	X
Paolo and Xavier $\rightarrow$	X	X

The radius of circle  $A$  is 1 unit. In  $\triangle PQR$ ,  $m\angle RPQ = 30^\circ$  and  $RP = 1$  unit.



Use the Connect Line tool to draw triangles in circle  $A$  that will show how  $\triangle PQR$  can be placed in the circle to illustrate  $\cos(\theta)$ , where  $\theta = \pm \frac{\pi}{6} \pm n\pi$  for  $n = 0$  and 1 for all values of  $x$  and  $y$ .

Reset
Add Point
Connect Line  $\rightarrow$

Select two (2) points to connect or press and drag to create and connect points.

### Geometric Sequences

9-3

Common ratio:  $r$

$a, ar, ar^2, ar^3, \dots$  Ex: 3, 6, 12, 24, 48, ...

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_5}{a_4} = \dots$$

Recursive formula:  $a_n = a$

$$a_n = a \cdot r^{n-1}, \quad n > 0$$

Explicit formula:  $a_n = a_1 \cdot r^{n-1}, \quad n \geq 1$

$$-2 \cdot -2 = r$$

8th term of  $-3, 6, -12, \dots$

$$a_8 = -3(-2)^{8-1} = -3(-2)^7 = \boxed{384}$$

Write an explicit formula for  $a_1 = 4, r = 0.1$

$$a_n = 4 \cdot (0.1)^{n-1}$$

Find the missing terms 972,  $\dots$ ,  $\dots$ ,  $\dots$ , 12,  $\dots$

$$a_1 = 972 \quad a_5 = 12$$

$$a_5 = a_1 r^{5-1} \rightarrow 12 = 972 r^4$$

$$\frac{12}{972} = \frac{972 r^4}{972}$$

$$\sqrt[4]{\frac{1}{81}} = r^4 \rightarrow \sqrt[4]{\frac{1}{81}} = r$$

$$\frac{1}{3} = r$$

### Geometric Series

9-5

$$a_1 + ar + ar^2 + \dots + ar^{n-1} \quad r \neq 1$$

$$\text{Sum: } S_n = \frac{a_1(1-r^n)}{1-r} \quad r \neq 1$$

Find the sum of  $-15 + 30 - 60 + 120 + \dots + 1920$

$$\text{Find } n \rightarrow a_n = a_1 r^{n-1} \quad S_8 = -15(1 - (-2)^8)$$

$$1920 = -15 r^{n-1} \quad = \frac{3825}{3} = 1275$$

$$-128 = -2^{n-1} \rightarrow 7 = n-1, \quad n = 8$$

Find the sum of  $\sum_{k=0}^n 4\left(\frac{1}{2}\right)^k$

$$a_1 = 4 \quad r = \frac{1}{2}, \quad n = 8$$

$$S_n = \frac{4(1 - (\frac{1}{2})^9)}{1 - \frac{1}{2}} = \boxed{8}$$

Series - Infinite

$$|r| < 1$$

$$S = \frac{a_1}{1-r}$$

converges

$$|r| \geq 1$$

no finite sum

diverges

Evaluate  $1 - \frac{1}{2} + \frac{1}{4} - \dots$

$$r = -\frac{1}{2}$$

$$|r| = \left| -\frac{1}{2} \right| < 1$$

$$\text{Sum} = \frac{1}{1 - (-\frac{1}{2})} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3}$$

converges

\*