

Write the polynomial function in standard form. Then classify it by degree and by number of terms.

1. $6x^5 - 2x^2 + 1 - 2x^5$

$6x^5 - 2x^5 - 2x^2 + 1 = 3x^5 - 2x^2 + 1$

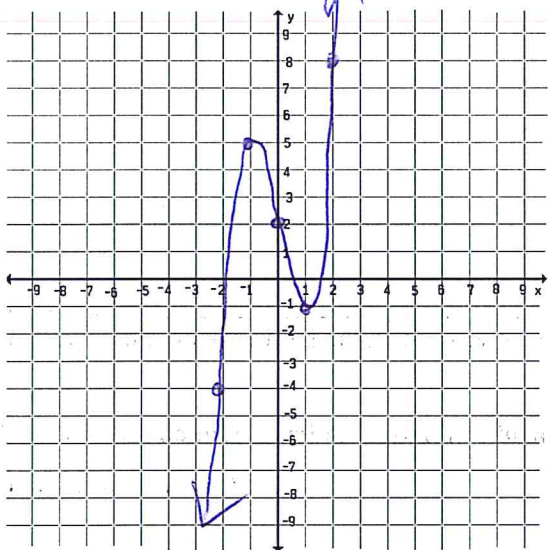
Determine the end behavior of the graph of the polynomial function. Then graph the function and name the relative minimum and relative maximum.

2. $y = 2x^3 - 5x + 2$

End Behavior: *down and up*

Relative max: 5
 $(-1, 5)$

Relative Min: -1
 $(1, -1)$



| x | y | |
|----|----|----------------|
| 0 | 2 | |
| 1 | -1 | $2 - 5 + 2$ |
| 2 | 8 | $16 - 10 + 2$ |
| -1 | 5 | $-2 + 5 + 2$ |
| -2 | -4 | $-16 + 10 + 2$ |

© 2005, Agnes Azzolino
 Permission is granted to duplicate as needed for nonprofit purposes.
 www.mathnstuff.com/gif9x9not.gif

Find the zeros of each function. State the multiplicity of each zero.

3. $y = (x + 2)(x - 3)^2$

$0 = (x + 2)(x - 3)^2$

$x + 2 = 0$

$x = -2$
m 1

$(x - 3) = 0$

$x = 3$
m 2

Divide using long division. Show all work.

4. $(x^3 + 2x^2 + 4x + 10) \div (x + 1)$

$$\begin{array}{r}
 x^2 + x + 3 + \frac{7}{x+1} \\
 \hline
 x^2 \overline{) (x+1) x^3 + 2x^2 + 4x + 10} \\
 \underline{-x^3 + x^2} \\
 x^2 + 4x \\
 \underline{-x^2 + x} \\
 3x + 10 \\
 \underline{-3x + 3} \\
 7
 \end{array}$$

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

5. $-1, 2, 6$

$P(x) = (x + 1)(x - 2)(x + 6)$
 $= (x^2 - x - 2)(x + 6)$
 $= x^3 + 6x^2 - x^2 - 6x - 2x - 12$

$P(x) = x^3 + 5x^2 - 8x - 12$

6. $-i, \sqrt{2}$
 $i, -\sqrt{2}$

$P(x) = (x + i)(x - i)(x - \sqrt{2})(x + \sqrt{2})$
 $= (x^2 - i^2)(x^2 - 2)$
 $= (x^2 + 1)(x^2 - 2)$
 $= x^4 - 2x^2 + 1x^2 - 2$

$P(x) = x^4 - x^2 - 2$

Find all the zeros of each function.

7. $y = x^3 + 8$

$0 = x^3 + 8$

$0 = (x+2)(x^2 - 2x + 4)$

$x+2=0$

$x = -2$

$x^2 - 2x + 4 = 0$

$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 4}}{2}$

$= \frac{2 \pm \sqrt{-12}}{2}$

$= \frac{2 \pm 2i\sqrt{3}}{2}$

$x = 1 \pm i\sqrt{3}$

8. $y = x^4 - 14x^2 + 45$

$0 = x^4 - 14x^2 + 45$

$0 = (x^2 - 5)(x^2 - 9)$

$x^2 - 5 = 0$

$x^2 = 5$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{5}, \pm 3$

For the following function, list the possible rational zeros. Then use Descartes' Rule of Signs to determine the number of positive and negative real roots. Then find all roots.

9. $y = x^4 - 8x^3 + 19x^2 - 32x + 60$

Neg: $y = x^4 + 8x^3 + 19x^2 + 32x + 60$
no changes

| | | | | | |
|-----|---|----|----|-----|-----|
| | 4 | 3 | 2 | 1 | 0 |
| 1/x | 1 | -8 | 19 | -32 | 60 |
| | | 1 | -7 | 12 | -20 |
| | 1 | -7 | 12 | -20 | 140 |

| | | | | | |
|-----|---|----|-----|-----|-----|
| 2/x | 1 | -8 | 19 | -32 | 60 |
| | | 2 | -12 | 14 | -36 |
| | 1 | -6 | 7 | -18 | 124 |

| | | | | | |
|-----|---|----|-----|-----|-----|
| 3/x | 1 | -8 | 19 | -32 | 60 |
| | | 3 | -15 | 12 | -60 |
| | 1 | -5 | 4 | -20 | 0 |

| | | | | | |
|-----|---|----|----|-----|----|
| 5/x | 1 | -8 | 19 | -32 | 60 |
| | | 5 | 0 | 20 | 0 |
| | 1 | 0 | 4 | 0 | 0 |

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60$

Number of real zeros:

Positive: 4 or 2 or 0

Negative: 0

$$\begin{aligned} -2 &= a+b+c \\ -4 &= -2a-2b-2c \\ 12 &= 8a+4b+2c \\ \hline 8 &= 6a+2b \\ 8 &= 6a+2(-2) \\ 8 &= 6a-4 \\ +4 & \\ \hline 12 &= 6a \quad a=2 \end{aligned}$$

Roots: 3, 5, 2i, -2i

OR at this point: $x^3 - 5x^2 + 4x - 20 = 0$
 $x^2(x-5) + 4(x-5) = 0$
 $(x^2+4)(x-5) = 0 \quad x=5, \pm 2i$

10. Determine the cubic function that is obtained from the parent function $y = x^3$ after the sequence of transformations.

a vertical stretch by a factor of 3; a reflection across the y-axis; and a horizontal translation 4 units right

$y = -3(x-4)^3$

11. What is $P(2)$ given that $P(x) = 3x^4 - x^3 + 2x^2 - 10$? Use synthetic division and the Remainder Theorem.

| | | | | | |
|-----|---|----|----|----|-----|
| 2/x | 3 | -1 | 2 | 0 | -10 |
| | | 6 | 10 | 24 | 48 |
| | 3 | 5 | 12 | 24 | 38 |

$P(2) = 38$

12. Write a polynomial function of degree 3 with rational coefficients and exactly one real zero. List all of the zeros of the function.

zeros: 3, i, -i

$f(x) = (x-3)(x-i)(x+i)$
 $= (x-3)(x^2+1)$

$f(x) = x^3 - 3x^2 + x - 3$

13. Find a polynomial function whose graph passes through (-1, -1), (0, 5), (1, 7). And (2, 17)

$-1 = a(-1)^3 + b(-1)^2 + c(-1) + d$
 $5 = a(0)^3 + b(0)^2 + c(0) + d \rightarrow 5 = d$
 $7 = a(1)^3 + b(1)^2 + c(1) + d$
 $17 = a(2)^3 + b(2)^2 + c(2) + d$

$$\begin{aligned} 2 &= 2-2+c \\ 2 &= c \\ y &= 2x^3 - 2x^2 + 2x + 5 \\ -6 &= -a+b-c \\ 2 &= a+b+c \\ 12 &= 8a+4b+2c \\ -6 &= -a+b-c \\ 2 &= a+b+c \\ -4 &= 2b \quad b = -2 \end{aligned}$$