# Section 9.5 Geometric Series

**geometric series**: the sum of the terms of a geometric sequence

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}$$

Finite

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a r^{n-1} \cdot r$$

$$-r S_n = -a_1 r + -a_1 r^2 + -a_1 r^3 + \cdots -a r^{n-1} + -a r^n$$

$$S_n - r S_n = a_1 - a_1 r^n \qquad S_n = \frac{a_1(1-r^n)}{(1-r)}$$

$$\frac{S_n(1-r)}{(1-r)} = \frac{a_1(1-r^n)}{(1-r)}$$

Infinite $|r| < 1$

**Converges**

$$S_n = \frac{a_1}{1-r}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$\left(\frac{1}{2}\right)^{1000}$ $9.33 \times 10^{-302}$ $\frac{1}{\text{GIANT}}$

Infinite $|r| > 1$

**Diverges**

does not have a finite sum $r = 2$

1, -2, +4, -8

Problem 1:

What is the sum of the geometric series?

a. 4 + 12 + 36 + 108 + 324 + 972 + 2916 $r=3$

$\frac{36}{12} = \frac{12}{4} = 3$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

* $a_n = a_1 r^{n-1}$

$2916 = 4(3^{n-1})$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$6 = n-1$

$7 = n$

$= \frac{4(1-3^7)}{1-3} = \frac{4(1-2187)}{-2}$

$1457.\overline{3}$

b. $\sum_{n=0}^{11} 3(-1.5)^n$ $a \cdot b^n$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$a_1 = 3(-1.5)^0 = 3 \cdot 1 = 3$

$11 - 0 = 11 + 1 = 12$

$r = -\frac{3}{2}$

$\frac{2}{5} \cdot \frac{3(1-(-\frac{3}{2})^{12})}{1+\frac{3}{2}} = \frac{6}{5}\left(1-\frac{3^{12}}{2^{12}}\right)$

$= -154.496$

Problem 2:

A game show is offering a prize of 1 cent on the first day, 3 cents on the second day, 9 cents on the third day, etc. What is the total amount of money earned from this prize in two weeks?

$1 + 3 + 9 + \cdots a_{14}$

$a_1$ $r=3$ $n=14$

$S_n = \frac{a_1(1-r^n)}{(1-r)} = \frac{1(1-3^{14})}{(1-3)}$

$-\frac{1}{2}(1-3^{14})$

$-\frac{1}{2}(1-4{,}782{,}969)$

$= 2{,}391{,}484$ cents

**$23,914.84**

Problem 3: Does the infinite series converge or diverge? If it converges, what is the sum?

a. $-5 - \frac{5}{2} - \frac{5}{4} - \frac{5}{8} - \frac{5}{16} - \cdots$

$\times\frac{1}{2}$ $\times\frac{1}{2}$

$r = \frac{-\frac{5}{16}}{-\frac{5}{8}} = \frac{-\frac{5}{8}}{-\frac{5}{4}}$

$-\frac{5}{16} \cdot -\frac{8}{5} = \frac{1}{2}$

$-\frac{5}{8} \cdot -\frac{4}{5} = \frac{1}{2}$

Infinite: converges $|r| < 1$; diverges $|r| > 1$

$r = \frac{1}{2}$ — converges

$S_n = \frac{a_1}{(1-r)} = \frac{-5}{1-\frac{1}{2}} = \frac{-5}{\frac{1}{2}} = -10$

b. $\frac{1}{4} - \frac{3}{8} + \frac{9}{16} - \frac{27}{32} + \cdots$

$\cdot\frac{3}{2}$ $\cdot\frac{3}{2}$ $\cdot\frac{3}{2}$

$\frac{-\frac{3}{8}}{\frac{1}{4}}$ $-\frac{3}{8} \cdot \frac{4}{1} = -\frac{3}{2}$

$\frac{\frac{9}{16}}{-\frac{3}{8}}$ $\frac{9}{16} \cdot -\frac{8}{3} = -\frac{3}{2}$

$r = -\frac{3}{2}$

$\left|-\frac{3}{2}\right| = \frac{3}{2} > 1$

diverges

c. $\sum_{n=0}^{\infty} 1(0.8)^n$

$r = 0.8$

$|0.8| < 1$

$ab^n$

$1 \cdot (0.8)^n$

$a_1 = 1$

$r = 0.8$

converges

$S_n = \frac{a_1}{1-r} = \frac{1}{1-0.8} = \frac{1}{0.2} = \frac{10}{2} = 5$