$$\frac{\left(\frac{t}{s} + \frac{s}{s}1\right)}{\left(\frac{t}{s} - \frac{s}{s}1\right)} = \frac{\frac{t+s}{\cancel{s}} \cdot \frac{\cancel{s}}{t-s}}{\frac{t-s}{s}}$$

$$\frac{\frac{h\,h}{r\,h} + \frac{r\,r}{h\,r}}{\frac{h\,h}{r\,h} - \frac{r\,r}{h\,r}} = \frac{\frac{h^2+r^2}{\cancel{rh}} \cdot \frac{\cancel{rh}}{h^2-r^2}}{\frac{h^2-r^2}{\cancel{rh}}}$$

$h^2 \neq r^2$

$r, h \neq 0$

Sec. 9.5 Geometric Series

Geometric series: the sum of a geometric sequence

Geometric sequence: starts with a term, a_1, and continues by multiplying each term by r, the common ratio.

Ex: 3, 6, 12, 24, 48, ...

$a_1 = 3$

$r = 2 = \frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24}$

Sum of a Finite Geometric Series

$a_1 r^0 + a_1 r^1 + a_1 r^2 + \cdots + a_1 r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

n = the number of terms

Sum of an Infinite Geometric Series

$|r| < 1 \rightarrow$ finite sum

$$S = \frac{a_1}{1-r}$$

converges to a number S as n gets very large

$|r| > 1$ series diverges

Summation Notation

$$\sum_{n=3}^{10} n^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$$

Problem 1: Find the sum.

a. $4 + 12 + 36 + 108 + 324 + 972 + 2916$

Finite: $S = \frac{a_1(1-r^n)}{(1-r)}$

$a_1 = 4$

$r = 3$

$n = 7$

$S = \frac{4(1-3^7)}{(1-3)} = \frac{4(1-2187)}{-2}$

$= -2(-2186) = 4{,}372$

b. $\sum_{n=0}^{11} 3(-1.5)^n$

$S = \frac{a_1(1-r^n)}{(1-r)}$

$a_1 = 3$

$r = -1.5$

$n = (11-0)+1$

$n = 12$

$= \frac{3(1-(-1.5)^{12})}{1--1.5}$

$= \frac{3(1-(1.5)^{12})}{1+1.5}$

≈ -154.50

Problem 2: Converge or Diverge?

a. $-5-\frac{5}{2}-\frac{5}{4}-\frac{5}{8}-\frac{5}{16}-\ldots$

$r=\left|\frac{1}{2}\right| \overset{?}{<} 1$ yes converges $S=\frac{a_1}{1-r}=\frac{-5}{1-\frac{1}{2}}=\frac{-5}{\frac{1}{2}}=\boxed{-10}$

b. $\frac{1}{4}-\frac{3}{8}+\frac{9}{16}-\frac{27}{32}+\ldots$

$r=\left|-\frac{3}{2}\right| \overset{?}{<} 1$ no diverges

c. $\sum_{n=0}^{\infty} 1(0.8)^n$

$S=\frac{a_1}{1-r}$

$a_1=1 \quad r=|0.8| \overset{?}{<} 1$ yes $=\frac{1}{1-0.8}=\frac{1}{0.2}=\boxed{5}$