Section 9.4 Arithmetic Series

series: the indicated sum of the terms of a sequence $a_1 + a_2 + a_3 + a_4 + \cdots a_n$

finite series: has a first and a last term

infinite series: continues without end

$$a_1 + a_2 + a_3 + a_4 + \cdots$$

Arithmetic series: a series whose terms form an arithmetic sequence

$a_1 + a_2 + a_3 + \ldots + a_n$ finite arithmetic series

Sum of a Finite Arithmetic Series

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

$$1 + 3 + 5 + 7 + 9$$

+2 +2 +2 −2

5

② 10

① 10

25

$10 \to 5$

$18 \to 9$

$5 \to \boxed{\frac{5}{2}}$

$$S_5 = \frac{5}{2}(1+9)$$

$$= \frac{5}{2} = 10 \quad = 25$$

Problem 1:

What is the sum of the finite arithmetic series
$\overset{a_1}{14} + 17 + 20 + 23 + \ldots + \overset{a_n}{\underset{a_{35}}{116}}$? $\rightarrow d=3$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{\boxed{n}}{2}(14 + 116) = \frac{35}{2}(130)$

$35(65)$

$2{,}275$

$a_n = a_1 + d(n-1)$

$116 = 14 + 3(n-1)$
$116 = 14 + 3n - 3$
$116 = 11 + 3n$
$105 = 3n$
$n = 35$

Problem 2:

There are 30 rows of seats in a large arena. The first row contains 10 seats. Each successive row increases by 3 seats. How many seats are in the last row? How many seats are there in all? $a_{30} = 97$ seats

$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$ $S_{30} = \frac{30}{2}(a_1 + a_{30}) = 15(10 + 97)$

$10, 13, 16, \ldots, 97$ $d = 3$ $15(107)$

$a_n = a_1 + d(n-1)$

$a_{30} = 10 + 3(30-1)$

$10 + 90 - 3 = 97$

535
107
$\boxed{1605 \text{ seats}}$

Summation notation

$\sum$ Sigma

Limits: the least and greatest values of *n* in the series

$3^2 + 4^2 + 5^2 + \cdots + 108^2$ ✓

$= \sum_{n=3}^{108} n^2$

Problem 3:

What is the summation notation for the series?

a. -19 + -14 + -9 + ... + 221 + 226

$+5 \quad +5 \quad +5$

a_3

$d = 5$

$a_1 = -19$

$a_n = 226$

$* \quad a_n = \boxed{a_1 + d(n-1)}$

$226 = -19 + 5(n-1)$

$226 = -19 + 5n - 5$

$226 = \boxed{\boxed{5}n - 24}$

$250 = 5n \qquad n = 50$

$5 - 24 \quad -19$

$3 \cdot 5 - 24 \quad -9$

$dn + (a_1 - d)$

$$\sum_{n=1}^{50} 5n - 24$$

-19

b. 20 + 18 + 16 + ... + -24 + -26

$-2 \quad -2$

$d = -2$

$* \quad a_n = a_1 + d(n-1)$

$-26 = 20 + -2(n-1)$

$-26 = 20 - 2n + 2$

$-26 = \boxed{-2n + 22}$

$-48 = -2n$

$24 = n$

a_n

$$\sum_{n=1}^{24} -2n + 22$$

$-2 \quad \boxed{+22} \quad 20$

Problem 4:

What is the sum of the series written in summation notation? arithmetic

a. $\sum_{n=1}^{27} (-2n + 1)$ linear

$S_n = \frac{n}{2}(a_1 + a_n)$

$\boxed{\begin{aligned} a_1 &= -2 \cdot 1 + 1 \\ &= -2 + 1 \\ &= -1 \end{aligned}}$ $\boxed{\begin{aligned} &a_{27} \\ &-2(27)+1 \\ &-54+1 \\ &-53 \end{aligned}}$

$S_{27} = \frac{27}{2}(-1 - 53)$

$= \frac{27}{\cancel{2}}(\cancel{-54}^{-27}) = \boxed{-729}$

b. $\sum_{n=1}^{5} (n^2 + 5)$

$= (1^2+5) + (2^2+5) + (3^2+5) + (4^2+5) + (5^2+5) = 6 + 9 + 14 + 21 + 30$

$\boxed{80}$

c. $\sum_{n=0}^{5} (10^n)$

$= 10^0 + 10^1 + 10^2 + 10^3 + 10^4 + 10^5$

$= 1 + 10 + 100 + 1000 + 10{,}000 + 100{,}000$

$= \boxed{111{,}111}$

Problem 5:

What is the sum of the series written in summation notation?

$\sum_{n=1}^{85} (n^2 + 4n + 3)$ graphing

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