Sec. 8.3 Rational Functions and Their Graphs

$\frac{a}{x-h}+k$

Rational

$f(x)=\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions.

D: all real numbers except those values for which $Q(x)=0$

D: $Q(x)\neq 0$

$y=\frac{x^2}{x^2+1}$

$x^2+1=0$
$-1\ \ -1$
$x^2=-1$
$x=\pm i$

continuous graph

H.A. $y=1$

$\frac{4}{5}$

$\frac{3.2}{2}$

$y=\frac{(x+3)(x+2)}{(x+2)}$

$y=x+3$
$x\neq -2$

removable discontinuity (hole)

D: $x+2\neq 0$
$x\neq -2$

$y=-2+3=1$
$(-2,1)$
→ Hole

$\frac{1004}{998}\to 1$

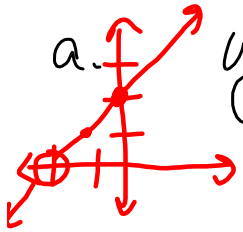
$y=\frac{x+4}{x-2}$

$x=2$

D: $x\neq 2$

Problem 1:

Removable Discontinuity

a. 

$y = \frac{x^2+4x+4}{x+2} = \frac{(x+2)(x+2)}{(x+2)}$

$y = x+2,\ x \neq -2$

D: $x \neq -2$

pts of discontinuity: $x = -2$ $y = -2+2 = 0$ $(-2,0)$ hole

x-int: none $0 = x+2$ $x = -2 \rightarrow$ not in domain

y-int: 2 $(0,2)$ $y = 0+2$ $y = 2$

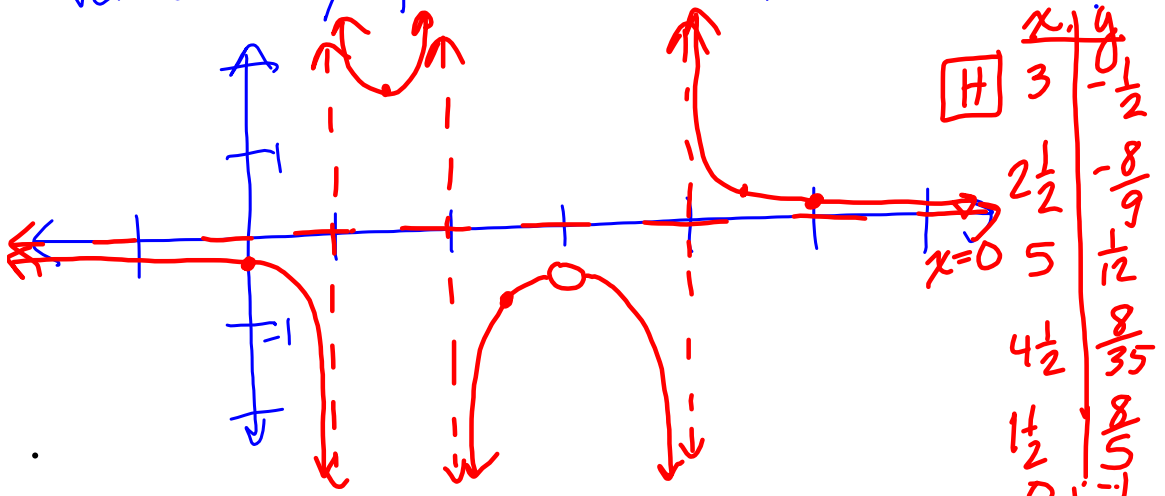
Problem 2: What are the vertical asymptotes?

$y = \frac{(x-3)^1}{(x^2-3x+2)(x^2-7x+12)}$

$(x-2)(x-1)(x-3)(x-4)$

D: $x \neq 1, 2, 3, 4$ Hole $x = 3$

Vertical asymptotes: $x = 1$, $x = 2$, $x = 4$



x	y
H 3	0
	$-\frac{1}{2}$
$2\frac{1}{2}$	$-\frac{8}{9}$
$x=0$ 5	$\frac{1}{12}$
$4\frac{1}{2}$	$\frac{8}{35}$
$1\frac{1}{2}$	$\frac{8}{5}$
	$-\frac{1}{8}$

$\frac{1}{4\cdot3\cdot1} = \frac{1}{12}$

$\frac{1}{(x-1)(x-2)(x-4)}$

$\frac{1}{2\cdot1\cdot-1} = -\frac{1}{2}$

$\frac{1}{\frac{1}{2}(-\frac{1}{2})(-\frac{5}{2})}$ $\frac{8}{5}$ 1.6

$\frac{1}{(\frac{3}{2})(\frac{1}{2})(\frac{3}{2})} = \frac{1}{\frac{9}{8}} = -\frac{8}{9}$

$\frac{1}{\frac{3}{2}(\frac{5}{2})(\frac{1}{2})}$ $\frac{8}{35}$

Horizontal Asymptotes

- occurs at $y = c$, where c is the value the function approaches as x gets large

* Plug in a "large" number, such as 1000 or 4,000,000

* $f(x) = \frac{P(x)}{Q(x)}$ → m = degree of $P(x)$; n = degree of $Q(x)$

$m < n$	$m = n$	$m > n$
ex: $y = \frac{x^{1000}}{x^{3}-1}$ (1000, 1,000,000,000)	ex: $\frac{3x^2-2}{5x^2+3}$ (3,000,000 / 5,000,000)	$y = \frac{x^3-1}{x}$ (1,000,000,000 = 1000)
$y = 0$	$y = \frac{3}{5}$	no horizontal asymptote
H.A.	$y = \frac{a}{b}$, where a is the leading coefficient of $P(x)$, b is the leading coefficient of $Q(x)$	

Problem 3: What are the horizontal asymptotes for the graph of

a. $y = \frac{x^2+1}{-3x+6}$ no horizontal asymptote (numerator has higher degree)

b. $y = \frac{-2x+6}{x-5}$ $y = -2$ (same degree)

c. $y = \frac{x-1}{x^2+4x+4}$ $y = 0$ (denominator has higher degree)

$f(x) = \frac{P(x) \rightarrow \text{degree } m}{Q(x) \rightarrow \text{degree } n}$

Vertical Asymptote (hole)

① Solve $Q(x) = 0$

- factors in $P(x)$ can be cancelled are holes; removable discontinuity
- factors that can't be cancelled give asymptotes non-removable discontinuity

Horizontal Asymptotes:

$m < n$ $\quad y = 0$

$m = n$ $\quad y = \frac{a_p}{a_q}$

$m > n$ $\quad$ no H.A.

Problem 4:

Graph $y = \frac{x^2+1}{-3x+6}$

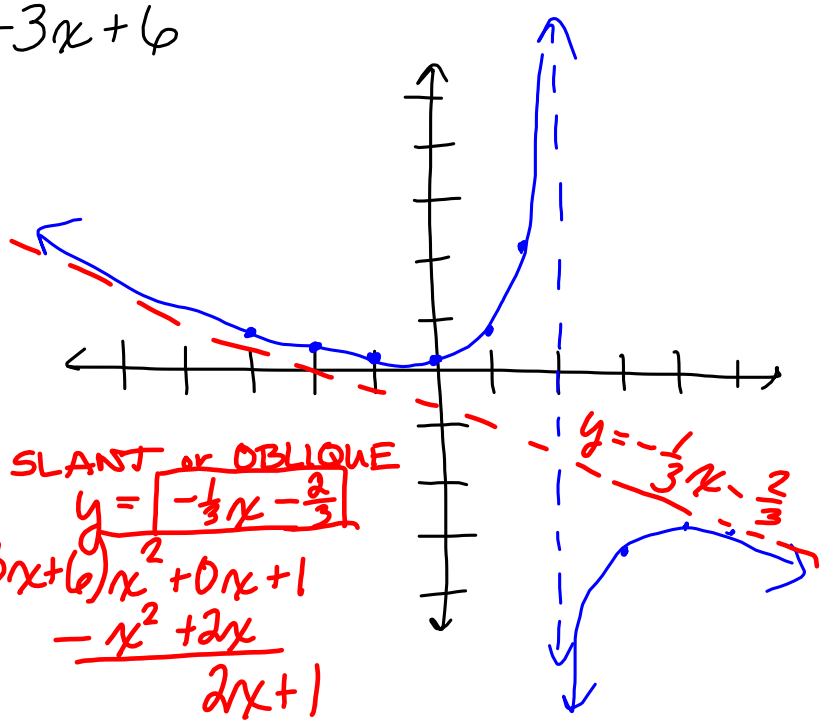
D: $x \neq 2$

V.A. $x = 2$

H.A. none

x	y
0	$\frac{1}{6}$
1	$\frac{2}{3}$
$\frac{3}{2}$	$\frac{13}{6}$
-1	$\frac{2}{9}$
-2	$\frac{5}{12}$
-3	$\frac{2}{3}$

x	y
3	$-\frac{16}{3}$
4	$-\frac{17}{6}$
5	$-\frac{26}{9}$



Problem 5:
Whole milk contains 3.7% fat. You want to add 2%-fat milk to 5 fl. oz. of whole milk to make 3%-fat milk. The function

$$y = \frac{(5)(0.037) + x(0.02)}{5+x}$$ gives the

percentage of fat in a new concentration after you add x fluid oz of the 2% milk. How much 2% milk must you add?

$$(5+x)\,0.03 = \frac{(0.185) + 0.02x}{5+x}(5+x)$$

$$0.15 + 0.03x = 0.185 + 0.02x$$
$$-0.02x \qquad\qquad -0.02x$$

$$0.15 + 0.01x = 0.185$$
$$-0.15 \qquad\quad -0.15$$

$$\frac{0.01x}{0.01} = \frac{0.035}{0.01} \cdot \frac{100}{100} = \frac{3.5}{1}$$

$$x = 3.5 \text{ oz.}$$