

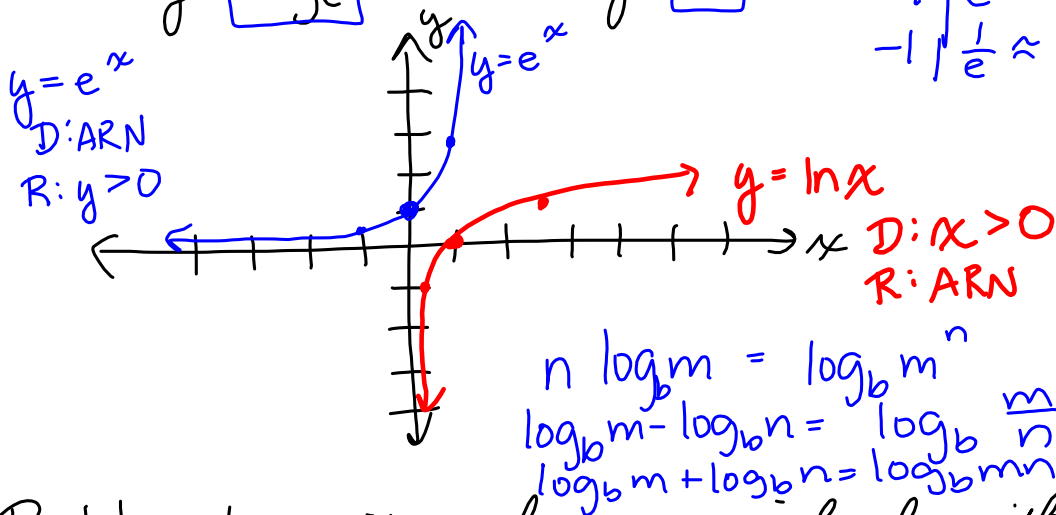
Sec. 7.6 Natural Logarithms

Natural Logarithmic Function

- inverse of $y = e^x$ - $y = \boxed{\log_e} x$ or $y = \boxed{\ln} x$

$$y = e^x$$

x	y
0	1
1	$e \approx 2.7$
-1	$\frac{1}{e} \approx 0.4$



Problem 1: write each as a single logarithm.

a. $3 \ln 2 - 3 \ln x$

$$\ln 2^3 - \ln x^3$$

$$\ln 8 - \ln x^3$$

$$\ln \frac{8}{x^3}$$

b. $2 \ln 3 + \ln 8$

$$\ln 3^2 + \ln 8$$

$$\ln 9 + \ln 8$$

$$\ln 72$$

c. $20 \ln x + 5 \ln y - 2 \ln z$

$$\ln x^{20} + \ln y^5 - \ln z^2$$

$$\ln x^{20} y^5 - \ln z^2$$

$$\ln \frac{x^{20} y^5}{z^2}$$

Problem 2: Solve

$$a. \ln \boxed{x} = 5$$

$$e^5 = x$$

$$x \approx 148.41$$

$$b. \ln \boxed{(6x+1)} > 0 = 3$$

$$\frac{e^3}{-1} = \frac{6x+1}{-1}$$

$$\frac{e^3 - 1}{6} = \frac{6x}{6}$$

$$x = \frac{e^3 - 1}{6} \approx 3.18$$

$$c. \ln \boxed{(2x+1)^2} > 0 = 6$$

$$\pm \sqrt{e^6} = \sqrt{(2x+1)^2}$$

$$\frac{\pm e^3}{-1} = \frac{2x+1}{-1}$$

$$\frac{\pm e^3 - 1}{2} = \frac{2x}{2}$$

$$x = \frac{\pm e^3 - 1}{2} \approx 9.54, -10.54$$

$$y = e^x \text{ or } y = b^x$$

$$D: \text{ARN}$$

$$R: y > 0$$

$$y = \ln x \text{ or } y = \log_b x$$

$$D: x > 0$$

$$R: \text{ARN}$$

Problem 3: Solve

$$a. e^{x+3} = 24$$

$$\frac{\log_e 24}{-3} = \frac{x+3}{-3}$$

$$(\log_e 24) - 3 = x$$

$$x \approx 0.18$$

$$\frac{\ln 24}{-3} = \frac{x+3}{-3}$$

$$(\ln 24) - 3 = x$$

$$x \approx 0.18$$

$$b. \frac{5e^{-3x}}{5} = \frac{45}{5}$$

$$e^{-3x} = 9$$

$$\frac{\ln 9}{-3} = \frac{-3x}{-3}$$

$$x = \frac{\ln 9}{-3} \approx -0.73$$

$$c. \frac{2e^{5x} - 8}{+8} = \frac{30}{+8}$$

$$\frac{2e^{5x}}{2} = \frac{38}{2}$$

$$e^{5x} = 19$$

$$\frac{\ln 19}{5} = \frac{5x}{5}$$

$$x = \frac{\ln 19}{5} \approx 0.59$$

Problem 4

Carbon-14 dating

$$t = \frac{\ln\left(\frac{N_f}{N_0}\right)}{-0.693} \times 5700$$

$$t = \frac{\ln 0.30}{-0.693} \times 5700 \approx 9903 \text{ yrs.}$$

$\frac{N_f}{N_0}$ = the percent of carbon 14 remaining

t = time in years

fossil 30% carbon-14 compared to living sample

Find its age