Sec. 7.5 Exponential and Logarithmic Equations

Problem 1: What is the solution of

a. $256^{2x} = 64$?

$4 \wedge 64$

$4 \wedge 16$

$4 \wedge 4$

$(4^4)^{2x} = 4^3$

$4^{8x} = 4^3$

$\frac{8x}{8} = \frac{3}{8}$

$\boxed{x = \frac{3}{8}}$

b. $27^{3x} = 81$

$(3^3)^{3x} = 3^4$

$3^{9x} = 3^4$

$\frac{9x}{9} = \frac{4}{9}$

$x = \frac{4}{9}$

c. $5^{x+1} = 25$

$5^{x+1} = 5^2$

$x + 1 = 2$

$-1 \quad -1$

$x = 1$

d. $5^{3x} = \frac{1}{125} = \frac{1}{5^3}$

$5^{3x} = 5^{-3}$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

Problem 2:

a. $6^{4x} = 512$

$\log_6 6^{4x} = \log_6 512$

$4x = \log_6 512$

$\frac{\log_6 512}{4} = \frac{4x}{4}$

$x = \frac{\log_6 512}{4}$

$\log 6^{4x} = \log 512$

$\frac{4x \log 6}{4 \log 6} = \frac{\log 512}{4 \log 6}$

$x = \frac{\log 512}{4 \log 6}$

$x = \frac{\log 512}{\log 6} \div 4 \approx 0.8704$

b. $5^{2x+7} - 1 = 8$

$+1 \quad +1$

$5^{2x+7} = 9$

$\log_5 9 = 2x + 7$

$-7 \quad -7$

$\log_5 9 - 7 = 2x$

$\frac{\log_5 9 - 7}{2} = \frac{2x}{2}$

$x = \frac{\log_5 9 - 7}{2} = \frac{\left(\frac{\log 9}{\log 5}\right) - 7}{2}$

≈ -2.8174

Problem 3:

a. $\log_{10}(5x+2)=2 \rightarrow 5x+2=10^2$

10 10

$10^2 = 5x+2$

$100 = 5x+2$

$-2 \quad -2$

$\frac{98}{5} = \frac{5x}{5}$

$19.6 = \frac{98}{5} = x$

$b^{\log_b x} = x$

$3^{\log_3 81} = 81$

b. $5 - 2\log x + \log 4 = 7$

$-5 \quad -5$

$-2\log x + \log 4 = 2$

$\log x^{-2} + \log 4 = 2$

$\log_{10} 4x^{-2} = 2$

$10^2 = 4x^{-2}$

$x^2 \cdot 100 = \frac{4}{x^2} \cdot x^2$

$\frac{100x^2}{100} = \frac{4}{100}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{25}}$

$x = \pm\frac{1}{5}$

$\frac{100}{4} = \frac{4x^{-2}}{4}$

$25 = x^{-2}$

$25 = \frac{1}{x^2}$

$\frac{1}{5}$ ✓ ~~$-\frac{1}{5}$~~

$\boxed{\frac{1}{5}}$

c. $\log 2x^2 - \log 5 = 1$

$5, -5$

$\log \frac{2x^2}{5} = 1$

$5 \cdot 10^1 = \frac{2x^2}{5} \cdot 5$

$\frac{50}{2} = \frac{2x^2}{2}$

$25 = x^2$

$\pm 5 = x$