Sec. 7.4 Properties of Logarithms

Product Property: $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^n = n \log_b m$

$\log_3 81^7 = 7 \log_3 81$

$\log_3 (3^4)^7 = 7 \log_3 3^4$

$\log_3 3^{28} = 7 \cdot 4$

$28 = 28$

$(2^6)^2 = 2^{6 \cdot 2} = 2^{12}$

$2 \log_2 (2^6)^2$

$\log_2 2^{6 \cdot 2} = 6 \cdot 2$

Why?

Let $x = \log_b m$ and $y = \log_b n$

$b^x = m$ and $b^y = n$ Def. of log.

$mn = b^x b^y$ Multiply

$mn = b^{x+y}$ Product Property of Exponents

$\log_b mn = x + y$ Def. of log.

$\log_b mn = \log_b m + \log_b n$ Substitution

$2^3 \cdot 2^2 = 2^{3+2}$

$\log_2 8 \cdot 4 = \log_2 8 + \log_2 4$

$8 \cdot 4 = 32$

$\log_2 32 = \log_2 2^3 + \log_2 2^2$

$5 = 3 + 2$

Problem 1: Write as a single logarithm.

a. $\log_4 5x + \log_4 3x$

$\log_4 (5x)(3x)$

$\log_4 15x^2 \rightarrow \log_4 15 + \log_4 x^2$

$\log_4 15 + 2\log_4 x$

b. $2\log_4 6 - \log_4 9$

$\log_4 6^2 - \log_4 9$

$\log_4 36 - \log_4 9$

$\log_4 \frac{36}{9} = \log_4 4 = 1$

c. $\log_4 32 - \log_4 2$

$\log_4 \frac{32}{2} = \log_4 16 = 2$

d. $6\log_2 x + 5\log_2 y$

$\log_2 x^6 + \log_2 y^5$

$\log_2 x^6 y^5$

Problem 2: What is each logarithm expanded?

a. $\log \frac{4x}{2y} = \log 4x - \log 2y$

$\log 4 + \log x - (\log 2 + \log y)$

$\log 4 + \log x - \log 2 - \log y$

b. $\log_9 \frac{x^4}{729} = \log_9 x^4 - \log_9 729$ (9, 81, 9)

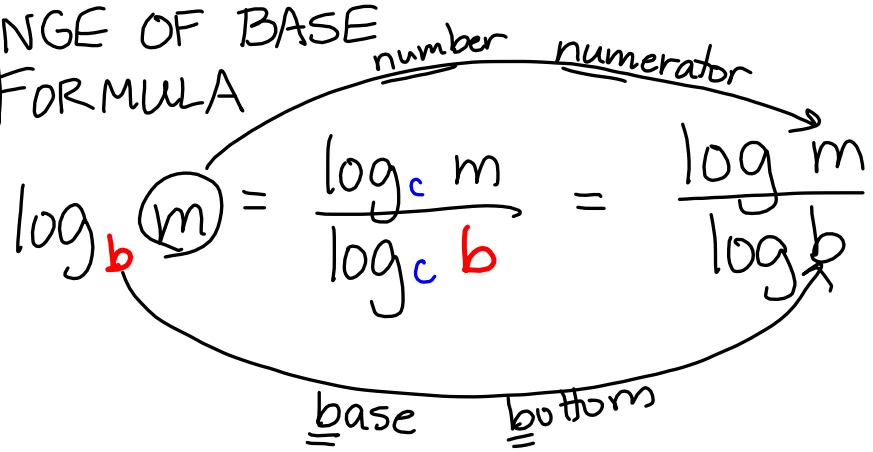
$4 \log_9 x - 3$

c. $\log_3 9x^5 = \log_3 9 + \log_3 x^5$

$2 + 5\log_3 x$

Problem 3: Find the value of each expression

* CHANGE OF BASE FORMULA



a. $\log_9 111 = \frac{\log 111}{\log 9} \approx 2.1433$

b. $\log_{81} 27 = \frac{\log_3 27}{\log_3 81} = \frac{3}{4}$

c. $\log_{216} 36 = \frac{\log_6 36}{\log_6 216} = \frac{2}{3}$

d. $\log_5 18 = \frac{\log 18}{\log 5} \approx 1.796$