Sec. 5.7 Binomial Theorem

Pascal's Triangle

$(a+b)^{n}$ n+1 terms

Row	
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 256 210 120 45 10 1

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)(a+b)$

$a^2 + ab + ab + b^2$

$a^2 + 2ab + b^2$

$(a+b)^3 = (a+b)\underbrace{(a+b)(a+b)}$

$(a+b)(a^2 + 2ab + b^2)$

$$\begin{array}{r} a^3 + 2a^2b + ab^2 \\ + \; a^2b + 2ab^2 + b^3 \\ \hline a^3 + 3a^2b + 3ab^2 + b^3 \end{array}$$

1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	4	1						
1	5	10	10	5	1					
1	6	15	20	15	6	1				
→ 1	7	21	35	35	21	7	1			
1	8	28	56	70	56	28	8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	1

Problem 1: Expand $(x+y)^7$

$$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Problem 2: Expand $(\overset{a}{x}+\overset{b}{2})^{\boxed{4}}$

number of terms:
$n+1$
$4+1=\boxed{5}$

1 4 6 4 1

$a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$1x^4+4x^3(2)^1+6x^2(2)^2+4x^1(2)^3+2^4$

exp.

$x^4+4x^3\cdot 2+6x^2\cdot 4+4x\cdot 8+16$

mult.

$\boxed{x^4+8x^3+24x^2+32x+16}$

Problem 3: $(3a - 7)^5$

1 5 10 10 5 1

$$(3a)^5 + 5(3a)^4(-7)^1 + 10(3a)^3(-7)^2$$

3^5a^5

$$+ 10(3a)^2(-7)^3 + 5(3a)^1(-7)^4 + 1\cancel{(3a)^0}(-7)^5$$

exp:

$$243a^5 + 5(81a^4)(-7) + 10(27a^3)(49) + 10(9a^2)(-343)$$
$$+ 5(3a)(2401) + (-16{,}807)$$

mult:

$$243a^5 - 2835a^4 + 13{,}230a^3 - 30{,}870a^2$$
$$+ 36{,}015a - 16{,}807$$

Problem 4:

4^{th} term : $(3x+5)^{5}$

1 5 10 10 5 1

x^5 x^4 x^3 x^2y^3

$10(3x)^2(5)^3$

$10(9x^2)(125)$

$11{,}250x^2$