Sec. 5.4 Dividing Polynomials

Long Division

$$32\overline{)4677}\quad 146\tfrac{5}{32}$$

-32

147

-128

197

-192

5

Polynomial Long Division

$$(x+1)\overline{)5x^2+2x+3}\quad 5x-3+\frac{6}{x+1}$$

$5x$

-3

$-5x^2 \mp 5x$

$-3x+3$

$+3x+3$

6

Problem 1: Is $x-2$ a factor of $P(x)=x^5-32$? If it is, write

a. $P(x)$ as a product of two factors.

x^4 $+4x^2$

x^3 $+2x$ $+8$

$$(x-2)\overline{)x^5+0x^4+0x^3+0x^2+0x-32}\quad x^4+2x^3+4x^2+8x+16$$

$-x^5+2x^4$

$2x^4+0x^3$

$-2x^4+4x^3$

$4x^3+0x^2$

$-4x^3+8x^2$

$8x^2+0x$

$-8x^2+16x$

$16x-32$

$-16x+32$

0

$P(x)=x^5-32$

$P(x)=(x-2)(x^4+2x^3+4x^2+8x+16)$

b. Is x^2+1 a factor of $3x^4-4x^3+12x^2+5$?

$$3x^2-4x+9+\frac{4x-4}{x^2+1}$$

$$\begin{array}{r} 3x^2 \\ (x^2+0x+1)\overline{)\ 3x^4-4x^3+12x^2+0x+5} \\ -4x \\ +9 \\ \underline{-3x^4+0x^3-3x^2} \\ -4x^3+9x^2+0x \\ \underline{+4x^3+0x^2+4x} \\ 9x^2+4x+5 \\ \underline{-9x^2+0x-9} \\ 4x-4 \end{array}$$

c. Long Division

$$(3x^3+9x^2+8x+4)\div(x\underset{\text{OPP}}{+2})$$

$$\begin{array}{r} 3x^2 \\ 3x\ (x+2)\overline{)\ 3x^3+9x^2+8x+4} \\ 2 \\ \underline{-3x^3+6x^2} \\ 3x^2+8x \\ \underline{-3x^2+6x} \\ 2x+4 \\ \underline{2x+4} \\ 0 \end{array}$$

	3	2	1	0
-2	3	9	8	4
Multiply ↗ / Add ↓		-6	-6	-4
	3	3	2	0

Quotient: $3x^2+3x+2$ R0

$P(-2)=0$

$(x+2)(3x^2+3x+2)=3x^3+9x^2+8x+4=P(x)=0$

$x=-2 \quad 3x^2+3x+2=0$

b^2-4ac

$$x=\frac{-3\pm\sqrt{9-4\cdot3\cdot2}}{2\cdot3}=\frac{-3\pm\sqrt{-15}}{6}$$

$$x=-2,\ \frac{-3\pm i\sqrt{15}}{6}$$

Problem 2: $(x^5 - 32) \div (x-2)$

Synthetic Division

a.

	5	4	3	2	1	0
2	1	0	0	0	0	−32
		2	4	8	16	32
	1	2	4	8	16	0

$\rightarrow P(2) = 0$

$Q: x^4 + 2x^3 + 4x^2 + 8x + 16$

b. Divide $4x^3 - 3x^2 + 2x - 3$ by $x - 1$.

	3	2	1	0
1	4	−3	2	−3
		4	1	3
	4	1	3	0

RO

$P(1) = 0$

$Q: 4x^2 + x + 3$

Zeros: $(x-1)(4x^2+x+3) = P(x) = 0$

$x = 1$ $\quad x = \dfrac{-1 \pm \sqrt{1-4\cdot4\cdot3}}{2\cdot4} = \dfrac{-1 \pm \sqrt{-47}}{8}$ ($1 - 48$)

$x = \dfrac{-1 \pm i\sqrt{47}}{8}$ or $-\dfrac{1}{8} \pm \dfrac{\sqrt{47}}{8}i$

The Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x-a$, then the remainder is $P(a)$.

Problem 3:

Given that $P(x) = x^4 + 6x^3 + 9x^2 + 3x - 3$ what is $P(4)$?

$P(4) = 4^4 + 6(4)^3 + 9(4)^2 + 3(4) - 3$

$= 256 + 384 + 144 + 12 - 3$

$= 796 - 3$

$= 793$ ✓

3: 16, 16, 96, 16, 256

2: 64, 6, 384

	4	3	2	1	0
4	1	6	9	3	-3
		4	40	196	796
	1	10	49	199	793 = P(4)

Ex: $4x^5 - x^4 - 7x^3 + 7x^2 - 18x + 18 = 0$

possible rational roots: $\pm \frac{\text{factors of constant}}{\text{factors of LC}}$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$4 \to 1, 2, 4 \to \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

$\to \pm\frac{1}{4}, \pm\frac{3}{4}, = \frac{9}{4}$

48
2 24

	5	4	3	2	1	0
1	1	-1	-7	7	-18	18
		1	0	-7	0	-18
3	1	0	-7	0	-18	0
		3	9	6	18	
-3	1	3	2	6	0	
		-3	0	-6		
	1	0	2	0		

$x = 1$

$x = 3$

$x = -3$

$x = i\sqrt{2}$

$x = -i\sqrt{2}$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm i\sqrt{2}$