Sec. 5.3 Solving Polynomial Equations

Polynomial Factoring Techniques

① GCF

② PS → Perfect Square Trinomial $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Diff. of 2 Squares $a^2 - b^2 = (a+b)(a-b)$

Sum of 2 Squares $a^2 + b^2 = (a+bi)(a-bi)$

③ PC

Sum of 2 Cubes: $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$\begin{array}{r} a^3 - a^2b + ab^2 \\ a^2b - ab^2 + b^3 \\ \hline a^3 + b^3 \end{array}$$

Difference of 2 Cubes: $a^3-b^3 = (a-b)(a^2+ab+b^2)$

④ Quadratic Trinomials:

ax^2+bx+c

product : ac sum b

$6x^2+11x-10 =$

$(3x-2)(2x+5)$

⑤ Factor by Grouping

$\underbrace{ax+ay}_{a(x+y)} + \underbrace{bx+by}_{b(x+y)} = (a+b)(x+y)$

What are the real or imaginary solutions?

Problem 1:

$4x^3 - 6x^2 = 4x$

$4x^3 - 6x^2 - 4x = 0$

$2x(2x^2 - 3x - 2) = 0$

$2x(2x+1)(x-2) = 0$

$2x = 0 \quad 2x+1 = 0 \quad x-2 = 0$

$x = 0 \quad x = -\frac{1}{2} \quad x = 2$

a.c. -4: $1 \cdot 4$, $2 \cdot 2$

$2x^2 + x - 4x - 2$

$x(2x+1) - 2(2x+1)$

$(2x+1)(x-2)$

Problem 2: Solve

$2x^3 = -54$

$\frac{2x^3}{2} + \frac{54}{2} = \frac{0}{2}$

$x^3 + 27 = 0$

$(x+3)(x^2 - 3x + 9) = 0$

$x + 3 = 0 \quad x^2 - 3x + 9 = 0$

$x = -3$

$x = \frac{3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$ $\quad 9 - 36$

$x = \frac{3 \pm \sqrt{-27}}{2}$ $\quad -27 < 9 < 3$

$x = \frac{3 \pm 3i\sqrt{3}}{2}$ $\quad \sqrt{-1 \cdot 3 \cdot 3 \cdot 3}$

$x = -3, \ \frac{3+3i\sqrt{3}}{2}, \ \frac{3-3i\sqrt{3}}{2}$

b. $8x^3 = 1$

$8x^3 - 1 = 0$

$(2x - 1)(4x^2 + 2x + 1) = 0$

$-2x$

$2x - 1 = 0$

$x = \frac{1}{2}$

$4x^2 + 2x + 1 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(4)(1)}}{8}$ (4 − 16)

$x = \frac{-2 \pm \sqrt{-12}}{8} < \frac{3}{4} < \frac{2}{2}$

$-\frac{2}{8} \pm \frac{2i\sqrt{3}}{8}$

$x = \frac{-2 \pm 2i\sqrt{3}}{8} = \frac{2(-1 \pm i\sqrt{3})}{2(4)}$

$-\frac{1}{4} \pm \frac{i\sqrt{3}}{4}$

$x = \frac{-1 \pm i\sqrt{3}}{4}$

$x = \frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}$

Problem 3: Solve.

$4x^3+3x^2-16x=12$

$4x^3+3x^2-16x-12=0$ Grouping / GCF

$x^2(4x+3)-4(4x+3)=0$

$(4x+3)(x^2-4)=0$

$4x+3=0 \quad x^2-4=0$

$x=\frac{-3}{4} \quad +4\ +4$

$\sqrt{x^2}=\pm\sqrt{4}$

$x=\pm 2$

$(x^2-4)(4x+3)=0$

$(x+2)(x-2)(4x+3)=0$

$x=-\frac{3}{4}, -2, 2$

Problem 4: Solve

$x^4-3x^2=4$

$x^4-3x^2-4=0$ (1·4, 2·2)

$(x^2+1)(x^2-4)=0$

① $x^2=-1 \quad x^2=4$

$x=\pm i \quad x=\pm 2$

② $(x+i)(x-i)(x+2)(x-2)=0$

$x=-i, i, -2, 2$

$u=x^2$

$u^2-3u-4=0$ (1·4, 2·2)

$(u+1)(u-4)=0$

$u=-1, 4$

$x^2=-1 \quad x^2=4$

$x=\pm\sqrt{-1} \quad x=\pm\sqrt{4}$

$x=\pm i \quad x=\pm 2$

$x^4-3\boxed{x^2}-4=0$

$\boxed{x^2}=\frac{3\pm\sqrt{9-4(1)(-4)}}{2}=-1,4$

b. $x^4+7x^2-18=0$

$(x^2+9)(x^2-2)=0$

$x^2=-9 \quad x^2=2$

$x=\pm\sqrt{-9} \quad x=\pm\sqrt{2}$

$x=\pm 3i \quad x=\pm\sqrt{2}$

$x=\sqrt{2}, -\sqrt{2}, -3i, 3i$