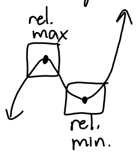
Sec. 5.2 Polynomials, Linear Factors, and Zeros

Vocabulary:

- Factor Theorem: The expression $x-a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function
- multiple zero: If a linear factor is repeated in the complete factored form of a polynomial, the zero related to that factor is a multiple zero
- multiplicity: the number of times a linear factor is repeated in the factored form of a polynomial
- relative maximum: the value of a function at an up to down turning point
- relative minimum: the value of a function at a down to up turning point



Key Concepts: Roots, Zeros, x-intercepts

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$

- $x-b$ linear factor of $P(x)$
- b root (or solution) of $P(x)=0$
- b zero of $y=P(x)$
- b x-intercept of the graph $y=P(x)$

Problem 1: What is the factored form of

x^3+x^2-12x

$x(x^2+x-12)$

$x(x+4)(x-3)$

Problem 2: What are the zeros of $y=(x-3)(x+4)(x-1)$? Graph the function.

$0=(x-3)(x+4)(x-1)$

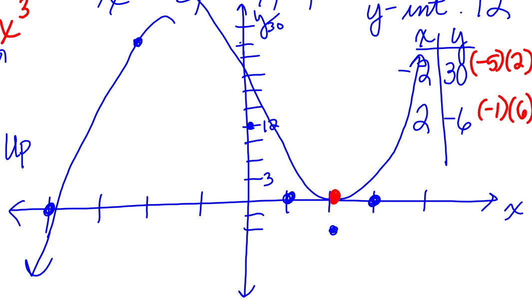
$x = 3, -4, 1$

y-int: 12 $\quad (-3)(4)(-1)$

eb: x^3

Down & Up

x	y	
-2	30	$(-5)(2)(-3)$
2	-6	$(-1)(6)(1)$



Problem 3: What is a cubic polynomial function in standard form with zeros, 1, −1, and 4?

$x = 1 \quad x = -1 \quad x = 4$

$x - 1 = 0 \quad x + 1 = 0 \quad x - 4 = 0$

$P(x) = (x-1)(x+1)(x-4)$

$(x^2 + \cancel{x} - \cancel{x} - 1)(x-4)$

$(x^2 - 1)(x - 4)$

$P(x) = x^3 - 4x^2 - x + 4$

Problem 4: What are the zeros of $f(x) = x^3 - 4x^2 + 4x$?

$= x(x^2 - 4x + 4)$

$= x(x-2)(x-2)$

Zeros: 0, 2 → multiplicity 2

0 ↓ multiplicity 1

How does the graph behave at these zeros?

2 : multiplicity 2 → graph touches the x-axis and turns around; resembles a parabola here; relative max/min, TP

0 : multiplicity 1 → graph looks close to linear where it crosses the x-axis