Sec. 4.8 Complex Numbers

Vocabulary

- imaginary unit: the complex number whose square is -1. i

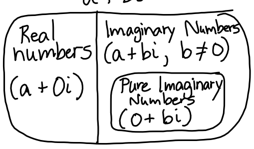
  $i^2 = -1$

  $i = \sqrt{-1}$

- imaginary number: any number in the form $a+bi$, where a and b are real numbers and $b \neq 0$.

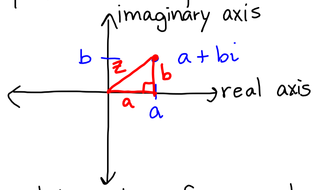
- complex number: set of numbers which contains imaginary numbers and real numbers together. $\underset{\text{real part}}{a} + \underset{\text{imaginary part}}{bi}$

Complex numbers
$a+bi$



- pure imaginary number: $a+bi$, where $a=0$, $b \neq 0$ $\rightarrow$ $0+bi$

- complex number plane:



$z^2 = a^2 + b^2$

$z = \sqrt{a^2+b^2}$

- absolute value of a complex number: distance from the origin in the complex plane $|a+bi| = \sqrt{a^2+b^2}$

Square Root of a Negative Real Number

$$\sqrt{-a} = \sqrt{-1 \cdot a} = \sqrt{-1} \cdot \sqrt{a} = i\sqrt{a}$$

Ex: $\sqrt{-5} = \sqrt{-1 \cdot 5} = i\sqrt{5}$

$\sqrt{-9} = \sqrt{-1 \cdot 9} = 3i$

Note: $(\cancel{\sqrt{}}{-5})^{\cancel{2}} = (i\sqrt{5})^2 = i^2(\sqrt{5})^2$

$= -1 \cdot 5 = -5$

- complex conjugates: number pairs of the form $a+bi$ and $a-bi$. The product is a real number.

$$(a+bi)(a-bi) = a^2 \boxed{\cancel{-abi+abi}} - b^2i^2$$

$-b^2(-1)$

$+b^2$

$= a^2 + b^2$

Note: $a^2 - b^2 = \underbrace{(a+b)(a-b)}_{\text{conjugates}}$

$a^2 + b^2 = \underbrace{(a+bi)(a-bi)}_{\text{complex conjugates}}$

Problem 1: Write $\sqrt{-24}$ using the imaginary unit i.

$$\sqrt{-24} = \sqrt{-1\cdot\cancel{2\cdot2}\cdot2\cdot3} = 2i\sqrt{6}$$

-1, 24; 2, 12; 2, 6; 2, 3

$$\sqrt{-1}\cdot\sqrt{2^2}\cdot\sqrt{2\cdot3}$$

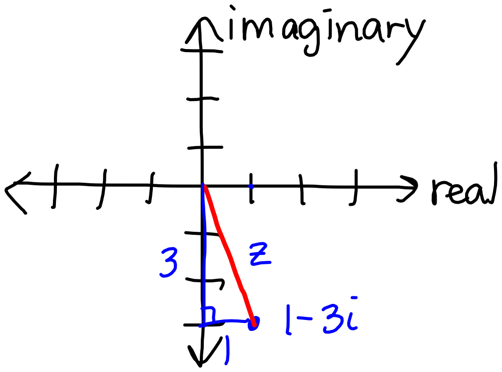
$$i\cdot2\cdot\sqrt{6}$$

$-1\,)\,-24$; $2\,)\,24$; $2\,)\,12$; $2\,)\,6$; $3\,)\,3$; 1

Problem 2: Graph and find the absolute value

a. $1-3i$

$$|1-3i| = \sqrt{a^2+b^2}$$
$$= \sqrt{1^2+(-3)^2}$$
$$= \sqrt{1+9}$$
$$= \sqrt{10}$$

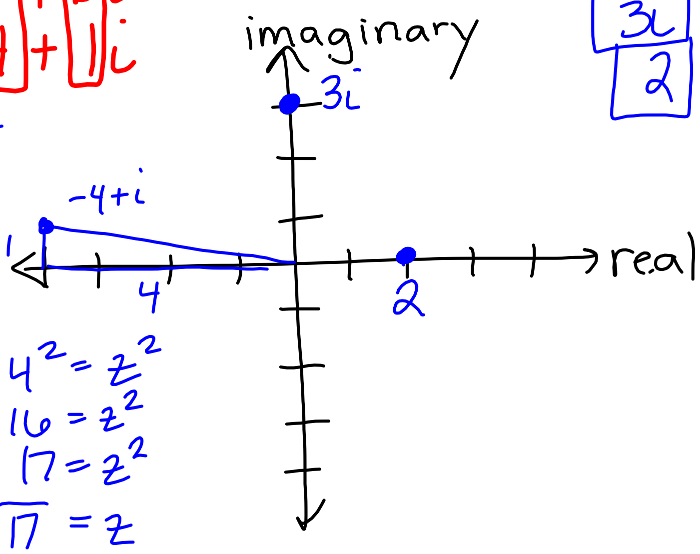


$$1^2+3^2 = z^2$$
$$1+9 = z^2$$
$$10 = z^2$$
$$\sqrt{10} = z$$

b. $-4+i$ $\boxed{a}+\boxed{b}i$ $\boxed{-4}+\boxed{1}i$

$\boxed{3i}$ $\boxed{2}$

$$|-4+i| = \sqrt{(-4)^2+(1)^2}$$
$$= \sqrt{16+1}$$
$$= \sqrt{17}$$



$$1^2+4^2 = z^2$$
$$1+16 = z^2$$
$$17 = z^2$$
$$\sqrt{17} = z$$

Problem 3: What is the sum or difference?

a. $(2+i)+(-3+3i) = -1+4i$

$2-3+i+3i$

b. $(5+i)+(5-i) = 10$

c. $(-6-2i)-(4+2i)$

$-6-2i-4-2i$

$-10-4i$

Problem 4: What is the product?

a. $-i(4-8i) = -4i \mp 8\cancel{i^2}$ $\quad i^2=-1$

$+8(-1)$

-8

$= -8-4i$

b. $(5-7i)(-4-3i)$

$\boxed{-20} - 15i + 28i \mp 21\cancel{i^2}$

$21(-1)$

$\boxed{-21}$

$-41+13i$

Problem 5: What is each quotient?

a. $\frac{(4-i)}{6i}\cdot\frac{i}{i} = \frac{4i-i^2}{6i^2} = \frac{4i-(-1)}{-6} = \frac{4i+1}{-6}$

$a+bi \quad = \frac{4i}{-6}+\frac{1}{-6} = -\frac{1}{6}-\frac{2}{3}i$

b. $\frac{7i}{(8+i)}\cdot\frac{(8-i)}{(8-i)} = \frac{56i+7\cancel{i^2}}{64\boxed{\cancel{-8i+8i}}+1\cancel{i^2}}$

$= \frac{7+56i}{65} = \frac{7}{65}+\frac{56}{65}i$

c. $\frac{(5+2i)}{(3-2i)}\cdot\frac{(3+2i)}{(3+2i)} = \frac{15+10i+6i\mp 4\cancel{i^2}}{9+\cancel{6i-6i}+4\cancel{i^2}}$

$= \frac{11+16i}{13} = \frac{11}{13}+\frac{16}{13}i$

Problem 6: What is the factored form?

a. $3x^2 + 12 = 3(x^2 + 4)$

$3(x + 2i)(x - 2i)$

b. $2x^2 + 50 = 2(x^2 + 25)$

$2(x + 5i)(x - 5i)$

Problem 7: What are the solutions of $-x^2 + 4x - 5 = 0$?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{\overset{16-20}{16 - 4(-1)(-5)}}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{\overset{i}{-}\overset{2}{4}}}{-2} = \frac{-4 \pm 2i}{-2}$$

$$= \frac{-4}{-2} \pm \frac{2i}{-2} = 2 \pm i$$