Sec. 4.7 The Quadratic Formula

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \frac{1}{7} + \frac{2}{7} = \frac{1+2}{7}$$

$$\frac{b}{2a} \cdot \frac{b}{2a} \qquad -\frac{c}{a} \quad -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{4ca}{4a \cdot a} + \frac{b^2}{4a^2}$$

$$\frac{1}{2} \cdot \frac{b}{a} \qquad \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(\frac{b}{2a}\right)^2 \qquad \sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Problem 1:

$$5x^2 - 2x = 2$$
$$-2 \quad -2$$
$$5x^2 - 2x - 2 = 0$$

$a = 5$

$b = -2$

$c = -2$

$\sqrt{3 \cdot 3}$

$\sqrt{9} = 3$

$\sqrt{3^2}$

$$x = \frac{2 \pm \sqrt{\overset{4+40}{4 - 4(5)(-2)}}}{2 \cdot 5}$$

$$x = \frac{2 \pm \sqrt{44}}{10} \quad 44 < 4 < 2, 2 ; 11$$

$$\frac{2 \pm \sqrt{2 \cdot 2 \cdot 11}}{10} \quad 2\sqrt{11}$$

$$x = \frac{\boxed{2} \pm \boxed{2}\sqrt{11}}{\boxed{10}}$$

$$\frac{\cancel{2}(1 \pm \sqrt{11})}{\cancel{2} \cdot 5}$$

$$x = \frac{1 \pm \sqrt{11}}{5} =$$

$$\frac{1}{5} \pm \frac{\sqrt{11}}{5}$$

Discriminant: $b^2 - 4ac$

$\pm\sqrt{0}$ $= 0$ one real solution

$\pm\sqrt{+}$ > 0 two real solution

$\pm\sqrt{-}$ < 0 no real solution (2 imaginary)

Problem 3: What is the number of real solutions of $-x^2 + 14x = 49$?

$b^2 - 4ac$

>0 2 real

$=0$ 1 real

<0 0 real (2 imag)

$$
\begin{array}{r}
-x^2 + 14x = 49 \\
\underline{\quad -49 \quad -49} \\
-x^2 + 14x - 49 = 0
\end{array}
$$

$b^2 - 4ac = 14^2 - 4(-1)(-49)$

$196 - 196$

0

1 real solution

b. $2x^2 - 3x + 7 = 0$

$b^2 - 4ac = 9 - 4(2)(7)$

$9 - 56$

-47

no real solution (2 imaginary)

Problem 4: A rocket is launched from the ground with an initial velocity of 150 ft/s. The function $h = -16t^2 + 150t$ models the height in feet of the rocket at time t in seconds. Will the rocket reach a height of 300 feet? Explain

$$h = -16t^2 + 150t$$
$$300 = -16t^2 + 150t$$
$$-300 \qquad\qquad -300$$
$$0 = -16t^2 + 150t - 300$$
$$b^2 - 4ac = 150^2 - 4(-16)(-300)$$
$$= 22{,}500 - 19{,}200$$
$$= 3300$$

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Yes, the discriminant is positive, so there are two real solutions for time t when the rocket's height is 300 ft.

Problem 5:
Write a quadratic equation with the given solutions.

a. $\frac{\boxed{-5}+\sqrt{17}}{4}$, $\frac{\boxed{-5}-\sqrt{17}}{4}$ $\quad \frac{\boxed{-b}\pm\sqrt{b^2-4ac}}{2a}$

$\frac{-b}{-1}=\frac{-5}{-1}$ $\quad \frac{2a}{2}=\frac{4}{2}$ $\quad b^2-4ac=17$

$b=5$ $\quad a=2$ $\quad 5^2-4(2)c=17$

$25-8c=17$
$-25 \quad\quad -25$

$\frac{-8c}{-8}=\frac{-8}{-8}$

$c=1$

$y=2x^2+5x+1$

b. $\frac{5+\sqrt{53}}{2}$, $\frac{5-\sqrt{53}}{2}$

$\frac{-b}{-1}=\frac{5}{-1}$ $\quad \frac{2a}{2}=\frac{2}{2}$ $\quad b^2-4ac=53$

$b=-5$ $\quad a=1$ $\quad (-5)^2-4(1)(c)=53$

$25-4c=53$
$-25 \quad\quad -25$

$\frac{-4c}{-4}=\frac{28}{-4}$

$c=-7$

$y=x^2-5x-7$

Problem 6: Solve $|x|=\begin{cases} x, & \text{if } x\geq 0 \\ -x, & \text{if } x<0 \end{cases}$

$|5-2x^2|=5$

$\boxed{5-2x^2}=5$
$-5 \quad\quad -5$

$\frac{-2x^2}{-2}=\frac{0}{-2}$

$x^2=0$

$x=0$

$-5+2x^2$

$\frac{\cancel{-1}(5-2x^2)}{\cancel{-1}}=\frac{5}{-1}$

$\boxed{5-2x^2}=-5$
$-5 \quad\quad -5$

$\frac{-2x^2}{-2}=\frac{-10}{-2}$

$x^2=5$

$\sqrt{x^2}=\pm\sqrt{5}$

$x=\pm\sqrt{5}$