Sec. 4.3 Modeling With Quadratic Functions

Problem 1:

A parabola contains the points $(0,1)$, $(-1,1)$ and $(1,5)$. What is the equation of this parabola in standard form?

$y = ax^2 + bx + c$

$1 = a(0)^2 + b(0) + c \rightarrow c = 1$

$1 = a(-1)^2 + b(-1) + c \rightarrow a - b + c = 1$

$5 = a(1)^2 + b(1) + c \quad a + b + c = 5$

$a - b = 0$

$a + b = 4$

$2a = 4$

$\frac{2a}{2} = \frac{4}{2}$

$a = 2$

sub. $a - b + 1 = 1$, $c = 1$

$1 \quad a + b + 1 = 5$

$2 + b = 4$

$-2 \quad -2$

$b = 2$

$(2, 2, 1)$

$y = 2x^2 + 2x + 1$

b. $(1,-2), (2,-4), (3,-4)$

$y = ax^2 + bx + c$

$-2 = a(1)^2 + b(1) + c \rightarrow a + b + c = -2$ ①

$-4 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -4$ ②

$-4 = a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = -4$ ③

② $4a + 2b + c = -4$

−① $-a - b - c = 2$

④ $3a + b = -2$

③ $9a + 3b + c = -4$

−① $-a - b - c = 2$

⑤ $8a + 2b = -2$

$3(1) + b = -2$

$3 + b = -2$

$-3 \quad -3$

$b = -5$

(-2) → $-6a - 2b = 4$

$\frac{2a}{2} = \frac{2}{2}$

$a = 1$

$a + b + c = -2$

$1 - 5 + c = -2$

$-4 + c = -2$

$+4 \quad +4$

$c = 2$

$(1, -5, 2)$

$y = x^2 - 5x + 2$

c. $f(-2) = 16, \quad f(0) = 0 \quad f(1) = 4$

$(-2, 16), (0,0), (1,4)$

$y = ax^2 + bx + c$

$16 = a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = 16$

$0 = a(0)^2 + b(0) + c \rightarrow c = 0$

$4 = a(1)^2 + b(1) + c \rightarrow a + b + c = 4$

$4a - 2b = 16$

$a + b = 4 \rightarrow$

$4 + b = 4$

$-4 \quad -4$

$b = 0$

$4a - 2b = 16$

$2a + 2b = 8$

$\frac{6a}{6} = \frac{24}{6}$

$a = 4$

$(4, 0, 0)$

$y = 4x^2$