

Sec. 3.2 Solving Systems Algebraically

Equivalent systems: have the same solutions.
We can multiply each side of one or both equations by the same number.

Problem 1: Solve using substitution.

$$\begin{array}{r} \overset{5}{1}x - \overset{-3}{2}y = -1 \\ \overset{1}{1}x + \overset{2}{1}y = 3 \\ \hline \overset{-1}{-1}x = \overset{-4}{-4} \end{array}$$

$$\textcircled{1} y = 3 - x = 3 - 1 = 2$$

$$\textcircled{2} 5x - 3(3 - x) = -1$$

$$\textcircled{3} 5x - 9 + 3x = -1$$

$$\begin{array}{r} 8x - 9 = -1 \\ +9 +9 \\ \hline 8x = 8 \end{array}$$

$$\textcircled{3} x = 1$$

$$\boxed{(1, 2)}$$

① Solve for x or y in one equation.

② Plug that value into the OTHER equation

③ Solve for one variable

④ Plug that variable into an equation and solve for other variable.

⑤ Write the solution as an ordered pair.

⑥ Check in BOTH equations

$$\begin{aligned}
 \text{b. } \boxed{x} + 3y &= 5 \checkmark \\
 -2x - 4y &= -5 \checkmark \\
 -2(5 - 3y) - 4y &= -5 \\
 -10 + 6y - 4y &= -5 \\
 -10 + 2y &= -5 \\
 +10 \quad \quad +10 & \\
 \hline
 2y &= 5
 \end{aligned}$$

$$\frac{2y}{2} = \frac{5}{2}$$

$$\checkmark y = \frac{5}{2} \quad \left(-\frac{5}{2}, \frac{5}{2}\right)$$

$$(-2.5, 2.5)$$

$$\begin{aligned}
 x + 3y &= 5 \\
 -3y \quad -3y & \\
 \hline
 \rightarrow \underline{x} &= 5 - 3y \checkmark \\
 x &= 5 - 3 \cdot \frac{5}{2}
 \end{aligned}$$

$$x = 5 - \frac{15}{2}$$

$$x = \frac{10}{2} - \frac{15}{2}$$

$$x = -\frac{5}{2}$$

$$\begin{aligned}
 \text{b. } 3x + 5y &= 13 \rightarrow 3x + 5(4 - 2x) = 13 \\
 2x + y &= 4 \\
 -2x \quad \quad -2x & \\
 \hline
 y &= 4 - 2x
 \end{aligned}$$

$$\begin{aligned}
 y &= 4 - 2x \\
 0 & \quad 4 - 2 \cdot 1 \\
 & \quad 4 - 2 \\
 & \quad 2
 \end{aligned}$$

$$(1, 2)$$

$$3x + 20 - 10x = 13$$

$$\begin{aligned}
 -7x + 20 &= 13 \\
 -20 \quad -20 & \\
 \hline
 -7x &= -7
 \end{aligned}$$

$$\frac{-7x}{-7} = \frac{-7}{-7}$$

$$x = 1$$

Problem 2: Solve by elimination.

$$\begin{array}{r} a. \quad 2x - 3y = 14 \\ \quad 4x + 3y = 46 \\ \hline \end{array}$$

$$\frac{6x}{6} = \frac{60}{6}$$

$$x = 10$$

$$(10, 2)$$

$$\begin{array}{r} 4 \cdot 10 + 3y = 46 \\ 40 + 3y = 46 \\ -40 \quad \quad -40 \\ \hline \end{array}$$

$$\frac{3y}{3} = \frac{6}{3}$$

$$y = 2$$

$$\begin{array}{l} b. \quad 9(7x + 5y = 2) \rightarrow 63x + 45y = 18 \\ \quad 5(8x - 9y = 17) \rightarrow 40x - 45y = 85 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \cdot 1 - 9y = 17 \\ 8 - 9y = 17 \\ -8 \quad \quad -8 \\ \hline -9y = 9 \\ y = -1 \end{array}$$

$$103x = 103$$

$$x = 1$$

$$(1, -1)$$

Problem 3.4

$$a. \begin{cases} 6x + 4y = 2 \\ -2(3x + 2y = -1) \end{cases}$$

$$\begin{array}{r} 6x + 4y = 2 \\ -6x - 4y = 2 \\ \hline 0 \neq 4 \\ \text{False} \end{array}$$

no solution
inconsistent

$$b. \begin{cases} 8(6x - 3y = 15) \\ 6(-8x + 4y = -20) \end{cases} \rightarrow \begin{array}{r} 48x - 24y = 120 \\ -48x + 24y = -120 \\ \hline 0 = 0 \end{array}$$

infinitely many solutions
consistent
dependent

0 = 0
True