Sec. 14.1 Trigonometric Identities

Pythagorean Identities:

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \cot^2\theta = \csc^2\theta$

③ $\tan^2\theta + 1 = \sec^2\theta$

Reciprocal Identies

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

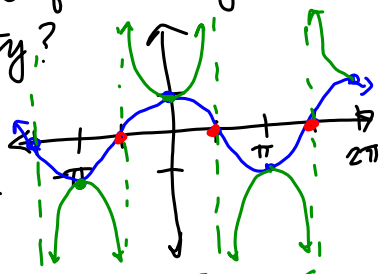
Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Domain of validity of an identity: set of values of the variable for which all expressions in the equation are defined

Problem 1:
What is the domain of validity of each trigonometric identity?

a. $\cos\theta = \frac{1}{\sec\theta}$



domain $\cos\theta \rightarrow$ ARN

domain $\sec\theta \rightarrow$ $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

odd multiples of $\frac{\pi}{2}$

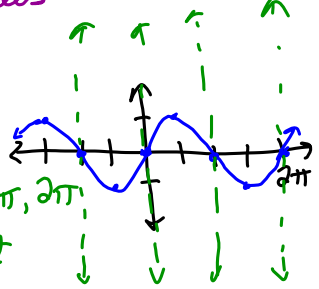
domain of validity:
all real numbers except for the odd multiples of $\frac{\pi}{2}$

$\theta \neq \frac{\pi}{2}n$: where n is the set of odd integers

b. $\sec\theta = \frac{1}{\cos\theta}$

domain of validity:
all real numbers except for the odd multiples of $\frac{\pi}{2}$

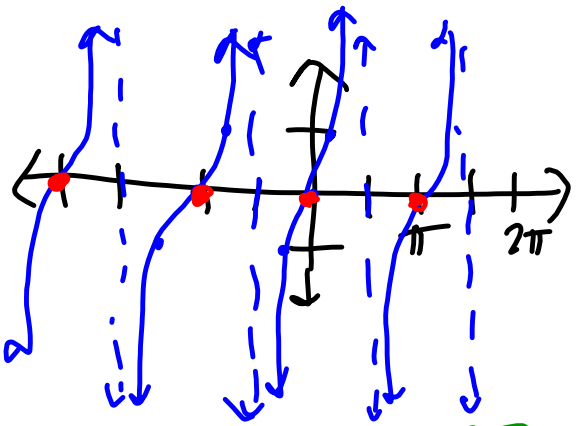
c. $\sin\theta = \frac{1}{\csc\theta}$



$\theta \neq 0, \pi, 2\pi$

DOV:
all real numbers except for the multiples of π

d. $\tan\theta = \frac{1}{\cot\theta}$



tan cot

$0 \to \frac{1}{0}$ undefined

$-\frac{\pi}{2}$ 0 $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π

$\frac{2\pi}{2}$ $\frac{3\pi}{2}$ $\frac{4\pi}{2}$

DOV:

all real numbers except all multiples of $\frac{\pi}{2}$

Problem 2: Verify the identity. What is the domain of validity?

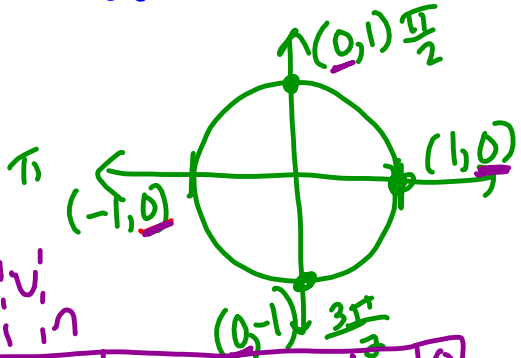
a. $(\sin\theta)(\cot\theta) = \cos\theta$

$\downarrow$

$(\sin\theta)\left(\frac{\cos\theta}{\sin\theta}\right) = \cos\theta$

$\cos\theta = \cos\theta$

DOV: all real numbers except multiples of π



y = 0

$\frac{1}{\sin\theta}$ → und

csc θ → mult. of π

sin θ	cos θ	csc θ	sec θ	tan θ	cot θ
y	x	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{y}{x}$	$\frac{x}{y}$
D: ARN	ARN	0, π, 2π... mult. π	ARN except odd mult. π/2	ARN except odd mult. π/2	ARN except mult. π

$\cot\theta = \frac{x}{y}$ → mult of π

$\tan\theta = \frac{y}{x}$

$\frac{1}{\sin\theta}$ → mult of π

b. $(\sec\theta)(\cot\theta) = \csc\theta$

$\theta \neq \frac{\pi}{2}$, $\theta \neq \pi$

$(\sec\theta)(\cot\theta)$

DOV: ARN except mult. of $\frac{\pi}{2}$

$= \left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \frac{1}{\sin\theta} = \csc\theta$ ✓

Problem: Verify the identity

a. $\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$-\cos^2\theta \quad -\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$\frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$

b. $1 + \cot^2\theta = \csc^2\theta$

$1 + \cot^2\theta = 1 + \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

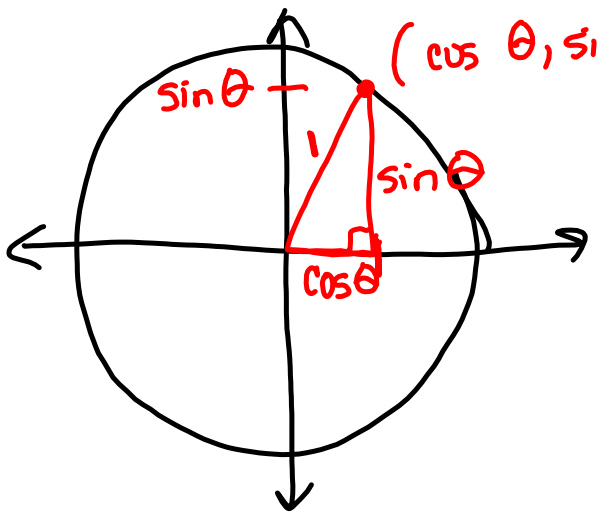
$\sin^2\theta + \cos^2\theta = 1$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$

$= \csc^2\theta$ ✓

Pythagorean Identity



$a^2 + b^2 = c^2$

$$\sin^2\theta + \cos^2\theta = 1$$

$(\sin\theta)^2 \rightarrow \sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Problem 4:

a. $(\sin^2\theta)(\sec\theta)+\cos\theta = \sec\theta$

$(\sin^2\theta)(\sec\theta)+\cos\theta$

$\sec\theta = \frac{1}{\cos\theta}$

$(\sin^2\theta)\left(\frac{1}{\cos\theta}\right)+\cos\theta$

$\frac{\sin^2\theta}{\cos\theta} + \cos\theta\left(\frac{\cos\theta}{\cos\theta}\right)$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$\frac{\sin^2\theta+\cos^2\theta}{\cos\theta}$ $\quad \sin^2\theta+\cos^2\theta = 1$

$\frac{1}{\cos\theta} = \sec\theta$ ✓

Problem 5: Simplify

$\frac{1+\cot^2\theta}{\cot^2\theta}$ $\quad 1+\cot^2\theta = \csc^2\theta$

$\frac{\csc^2\theta}{\cot^2\theta} = (\csc^2\theta)\left(\frac{1}{\cot^2\theta}\right)$

$\left(\frac{1}{\sin^2\theta}\right)(\tan^2\theta)$

$\left(\frac{1}{\sin^2\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$\frac{1}{\cos^2\theta}$

$= \boxed{\sec^2\theta}$