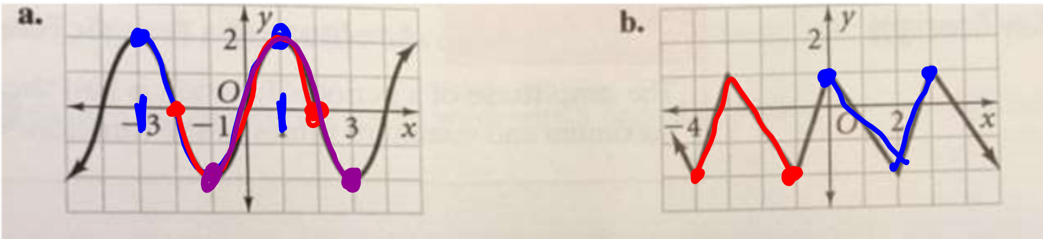
13.1 Exploring Periodic Data

Periodic function - repeats a pattern of y-vales (outputs) at regular intervals.
Cycle - 1 complete pattern. A cycle may begin at any point on the graph of the function.
Period - the horizontal length of 1 cycle, - in terms of x-values.

Example 1: Identifying Cycles and Periods

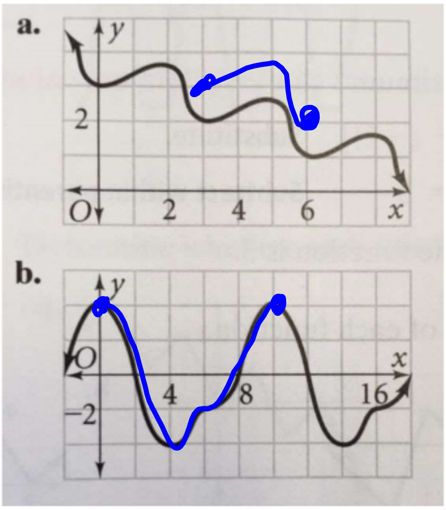
Analyze the periodic function. a) Identify 1 cycle in two different ways. b) Determine the period of the function.



$1-(-3)=4$ units $\quad$ $3-0=3$ units

Example 2: Identifying Periodic Functions

Determine whether each function is or is not periodic. If it is, find the period.



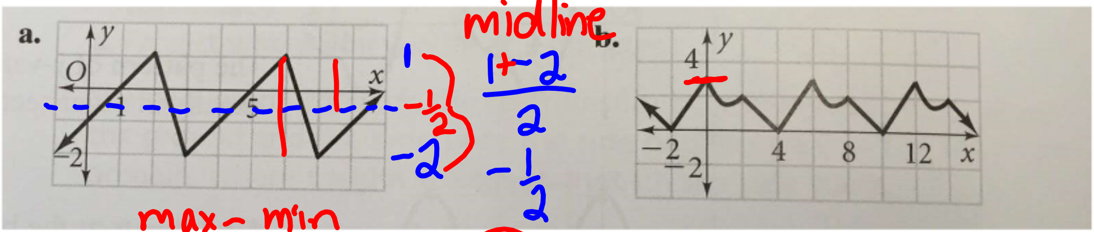
not periodic

periodic
9 units

$9-0$

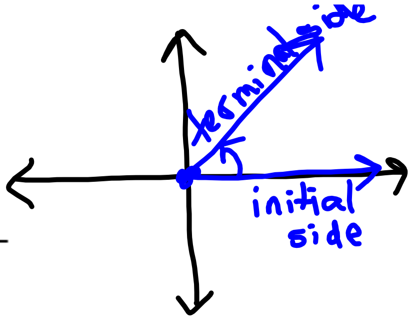
Amplitude - of a periodic function measures the amount of variation in the y-values. To find the amplitude:

Example 3: Find the Amplitude of the periodic function.



amp: $\frac{1-(-2)}{2} = \frac{3}{2}$

amp: $\frac{max_y - min_y}{2} = \frac{3-0}{2} = \frac{3}{2}$



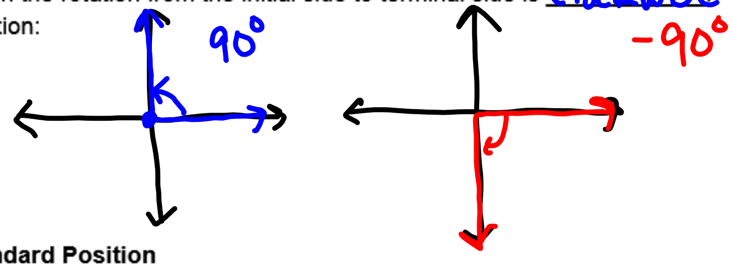
13.2 Angles (day 1)

An **angle in standard position** has:

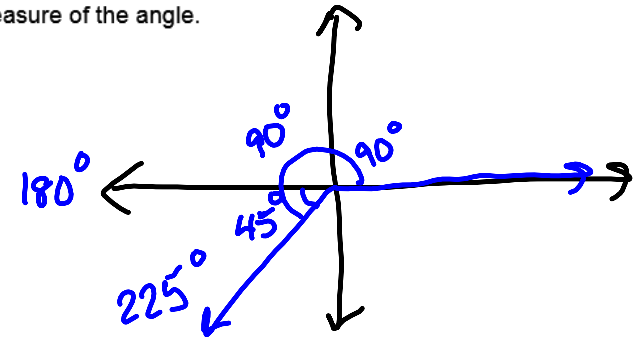
- vertex is at the origin
- one ray is on the x-axis

Initial side - the ray on the positive x-axis
Terminal side - the other ray of the angle
The **measure of an angle in standard position** is the amount of rotation from the initial side to the terminal side.
The measure of an angle is **positive** when the rotation from the initial side to terminal side is counterclockwise
The measure of an angle is **negative** when the rotation from the initial side to terminal side is clockwise

Sketch and angle in standard position:



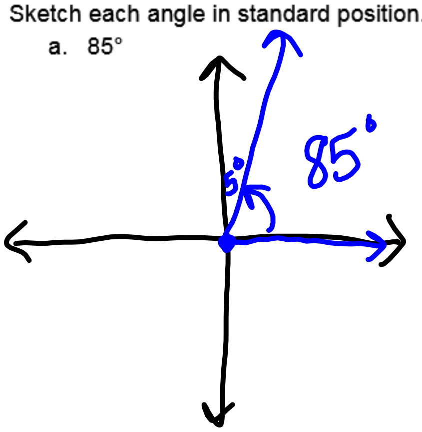
Example 1: Measuring an Angle in Standard Position
Find the measure of the angle.



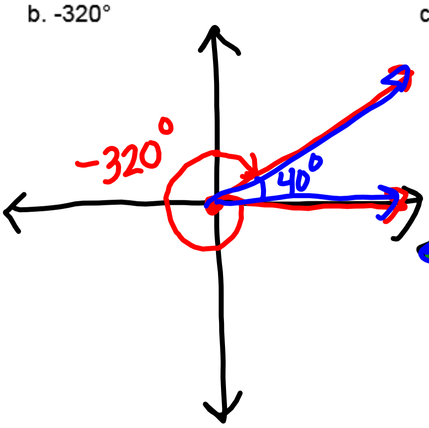
90+90+45

Example 2: Sketching an Angle in Standard Position
Sketch each angle in standard position.

a. 85°

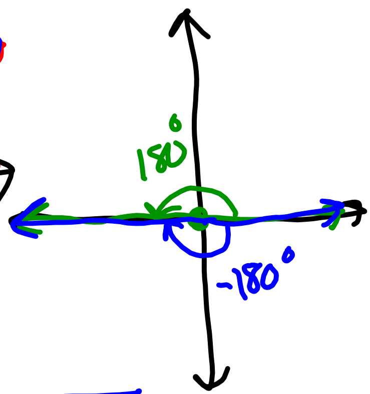


b. -320°



-320 + 360
40°

c. 180°



180° - 360°
-180°

Coterminal Angles - Two angles in standard position are coterminal if they have the same terminal side. For any given angle, there are infinitely many coterminal angles. To find a coterminal angle, **add or subtract 360°.**

Example 3: Finding Coterminal Angles

a. Find a positive angle and a negative angle that are coterminal with 198°.

$198^\circ + 360^\circ = 558^\circ$

$198^\circ - 360^\circ = -162^\circ$

b. Are the angles with measure 40° and 680° coterminal? Explain.

40° 680°. not coterminal. the difference is not a multiple of 360°

$680 - 40 = \frac{640}{360} = \times$

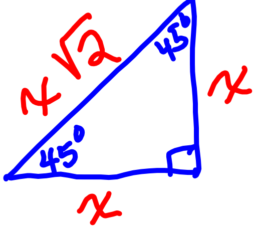
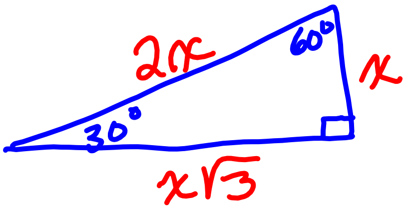
c. Find the measure of an angle between 0° and 360° coterminal with 385°.

$385^\circ - 360^\circ = 25^\circ$

d. Find the measure of an angle between 0° and 360° coterminal with -356°.

$-356^\circ + 360 = 4^\circ$

Recall: Special Right Triangles

45°-45°-90° Let a leg be x.	30°-60°-90° Let the shorter leg be x.
	

Find the missing side lengths in each 45°-45°-90° triangle. Rationalize any denominators.

1. hypotenuse = 1 inch

$x\sqrt{2} = 1$; $\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$; $x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$; $x = \frac{\sqrt{2}}{2}$

2. leg = 2 cm

$2 = x$; $x = 2$; $2\sqrt{2}$

3. hypotenuse = √3 ft

$\sqrt{3} = x\sqrt{2}$; $\frac{x\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$; $x = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$; $x = \frac{\sqrt{6}}{2}$

Find the missing side lengths in each 30°-60°-90° triangle. Rationalize any denominators.

4. shorter leg = 3 inch

$3 = x$; $2x = 6in$; $x\sqrt{3} = 3\sqrt{3}in$

5. longer leg = 1 cm

s: $x = \frac{\sqrt{3}}{3}$ cm

l: $x\sqrt{3} = 1$; $\frac{x\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$; $x = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$; $x = \frac{\sqrt{3}}{3}$

h: $2x = \frac{2\sqrt{3}}{3}$ cm ; $2 \cdot \frac{\sqrt{3}}{3} =$

6. hypotenuse = 1 ft

s: $x = \frac{1}{2}$ ft

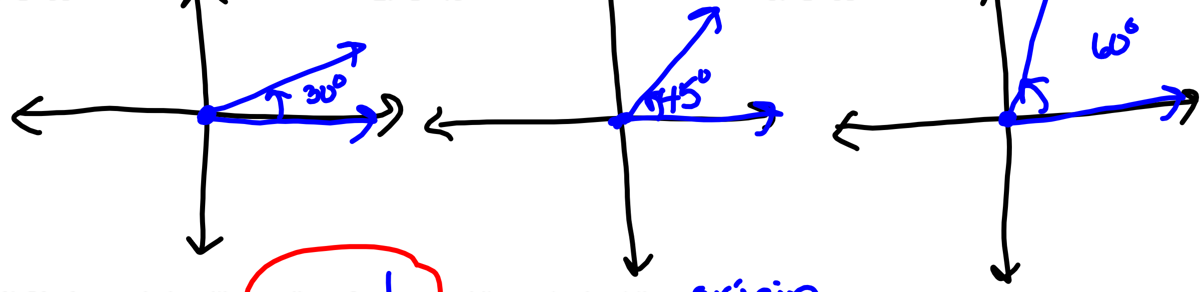
l: $x\sqrt{3} = \frac{1}{2} \cdot \sqrt{3}$; $\frac{\sqrt{3}}{2}$ ft

h: $\frac{2x}{2} = \frac{1}{2}$; $x = \frac{1}{2}$

13.2 The Unit Circle (day 2)

Warm Up: Sketch the angle in standard position:

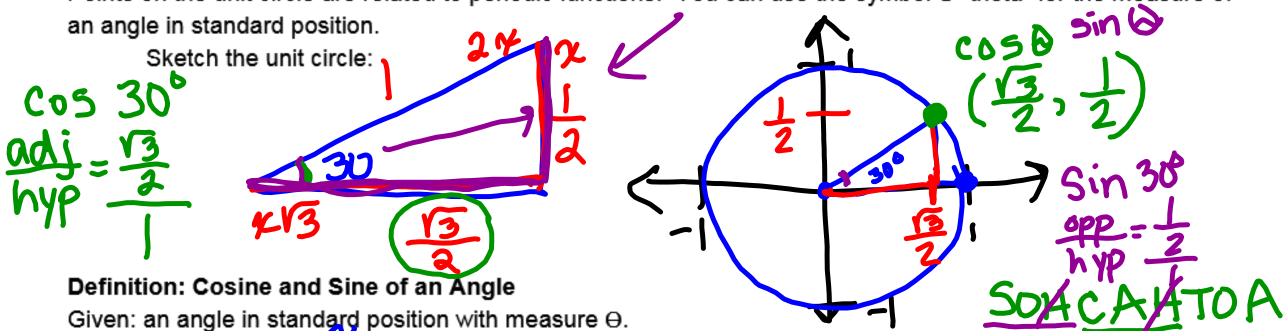
1. $\Theta = 30°$ 2. $\Theta = 45°$ 3. $\Theta = 60°$



Unit Circle - a circle with a radius of 1 and its center is at the origin

Points on the unit circle are related to periodic functions. You can use the symbol Θ "theta" for the measure of an angle in standard position.

Sketch the unit circle:



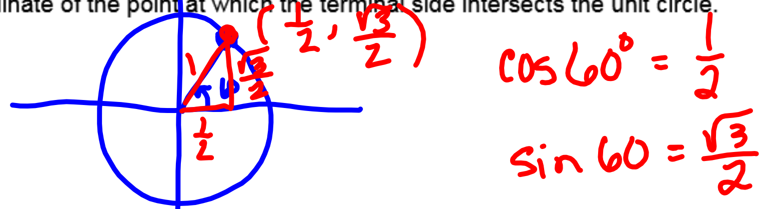
Definition: Cosine and Sine of an Angle

Given: an angle in standard position with measure Θ.

The **cosine of Θ** is the x -coordinate of the point at which the terminal side intersects the unit circle.

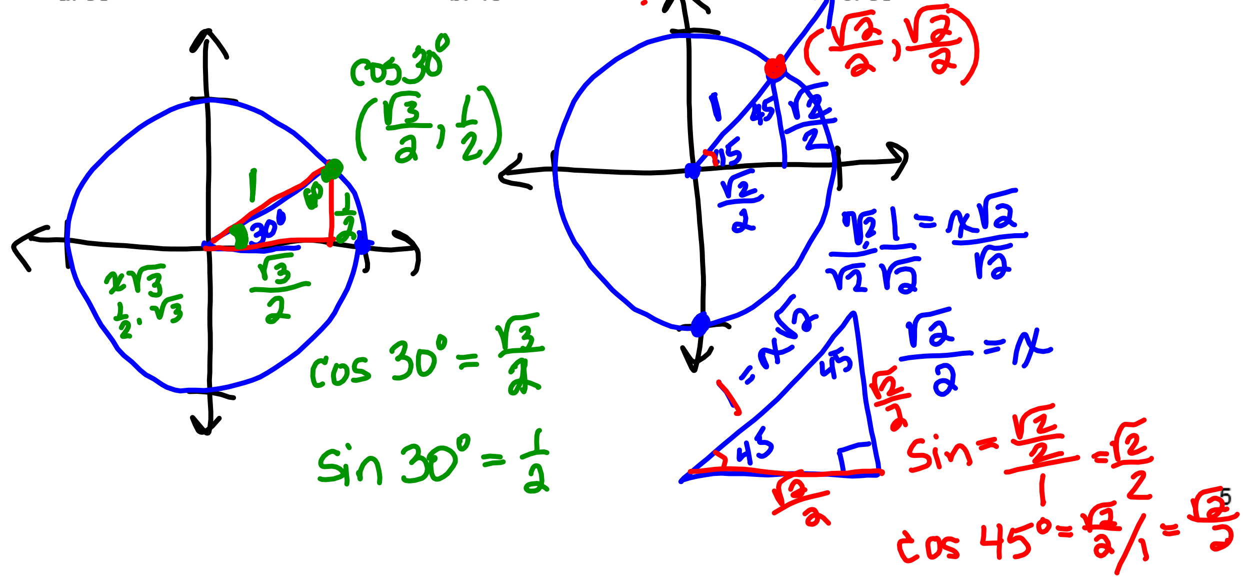
The **sine of Θ** is the y -coordinate of the point at which the terminal side intersects the unit circle.

Sketch:



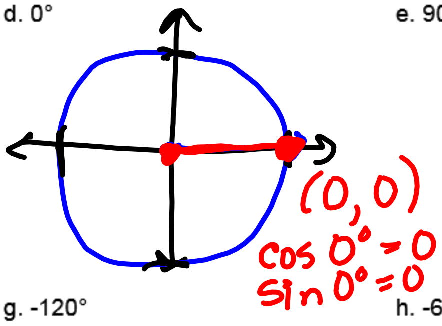
Example 4&5: Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

a. 30° b. 45° c. 60°

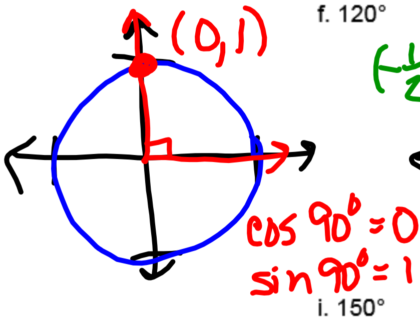


Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

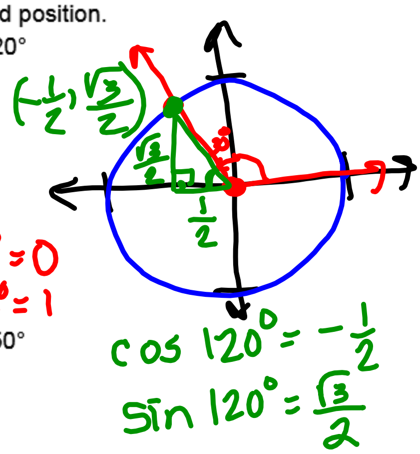
d. 0°



e. 90°



f. 120°



g. -120°

h. -60°

i. 150°

Practice Problems:

Calculator Needed: For angles that are not a multiple of 30° or 45°, you will need your calculator. Find cosϴ and sinϴ.

1. ϴ=32°

2. ϴ=-210°

3. ϴ=-10°

Find a positive and negative coterminal angle for the given angle.

4. 3. ϴ=400°

5. ϴ=-125°

6. 3. ϴ=-57°

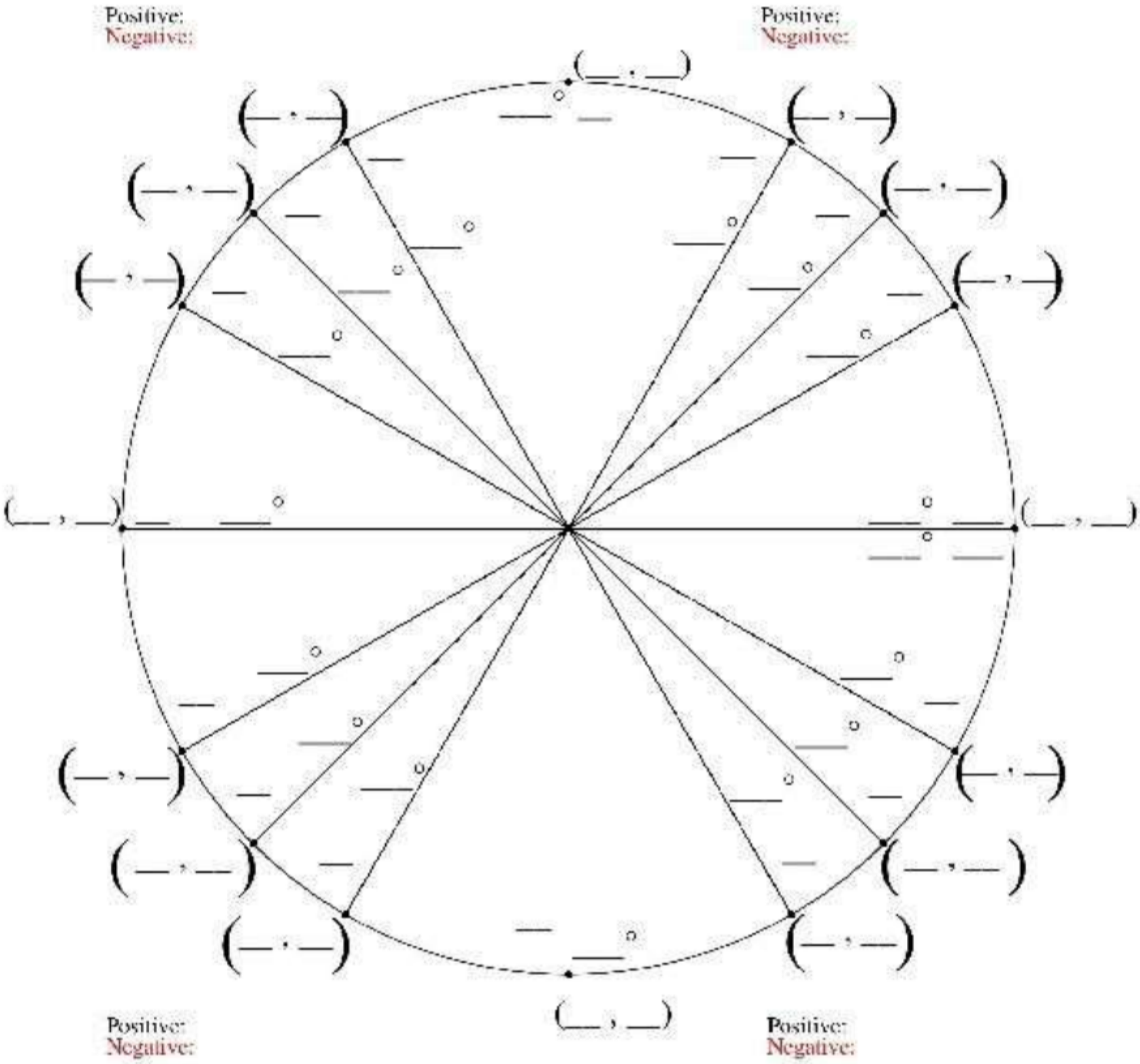
In which quadrant, or on which axis, does the terminal side of each angle lie? Sketch the angle to help you.

7. 210°

8. -60°

9. 270°

Fill in The Unit Circle



13.3 Radian Measure (Day 1)

Warm Up: Find the circumference of a circle with the given radius or diameter. Round your answer to the nearest tenth.

1. radius 4 in

2. diameter 70 m

Central angle - an angle whose vertex is the ___________ of a circle

sketch:

Intercepted arc - the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

Radian - when the intercepted arc equals the radius, the measure of the angle is 1 radian

sketch:

- The circumference of a circle is _________. Thus there are _______________ in any circle.
- Since 2π radians = ________°, then π radians = ________°
- Thus you can use this proportion to convert between degrees and radians.

Example 1: Use a proportion

a. Find the radian measure of 60°.

b. Find the degree measure of $\frac{5\pi}{2} radians$

Converting between Radians and Degrees

- To convert degrees to radians, multiply by __________
- To convert radians to degress, multiply by __________

Example 2: Using Dimensional Analysis

Convert the angle to degrees. Round to the nearest degree.

a. $-\frac{3\pi}{4} radians$ b. $\frac{\pi}{2} radians$ c. $2 radians$

Convert the angle to radians. Round to the nearest hundredth.

d. 27° e. 225° f. 150°

Example 3: Find the exact values of cosΘ and sinΘ for each angle measure.

Step 1: Convert to degrees.

Step 2: Draw the angle. The terminal side is the hypotenuse.

Step 3: Complete the right triangle. Draw a leg to the ***x-axis***.

Step 4: State the cosΘ and sinΘ.

a. $\frac{\pi}{4} radians$ b. $\frac{\pi}{6} radians$

c. $\frac{\pi}{2} radians$ d. $\frac{5\pi}{6} radians$

13.3 Arclength (Day 2)

Warm Up: Convert the angle to degrees. Draw the angle in standard position, as well as the corresponding right triangle. Then find the exact values for cosΘ and sinΘ.

$\theta = \frac{\pi}{3}$

You can find the length of an intercepted arc by using the proportion:

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure Θ (in *radians*), the length *s* of the intercepted arc is:

Example 4: Finding the Length of an Arc

Find the length of the intercepted arc to the nearest tenth. Sketch a diagram!

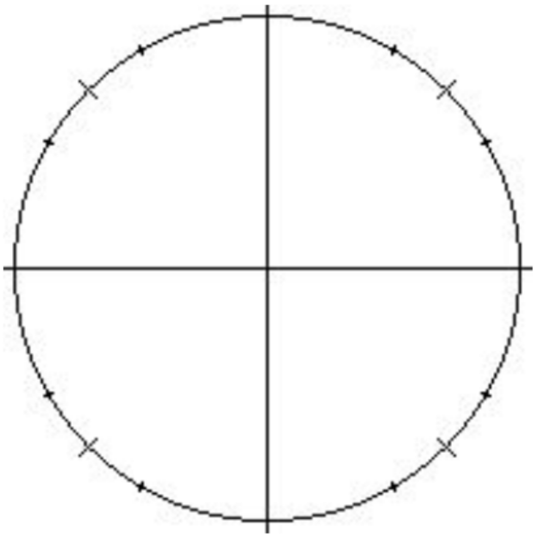
a. Given: A circle of radius 3 in, $\theta = \frac{5\pi}{6}$.

b. Given: A circle of radius 5m, $\theta = \frac{2\pi}{3}$.

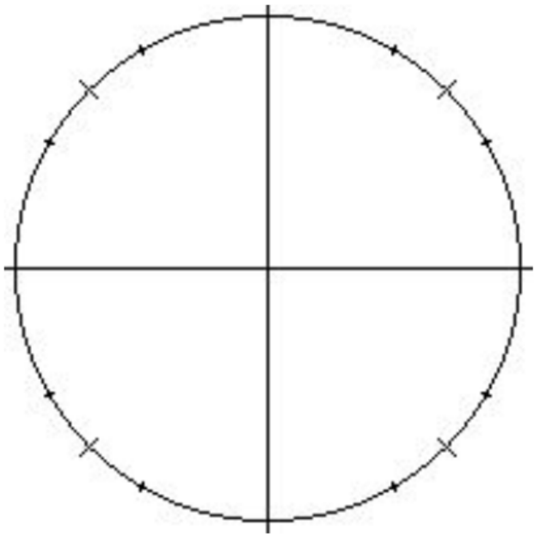
The Unit Circle: radius = ________

If you know quadrant 1, you can derive quadrants 2, 3, 4 by symmetry. Thus, let's study quadrant 1.

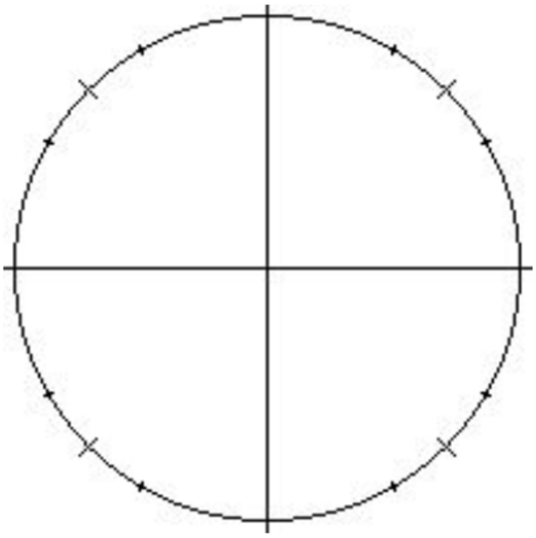
$\Theta=30°$



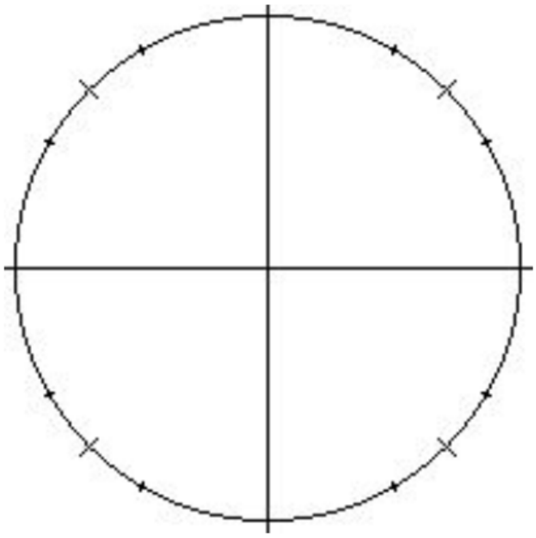
$\Theta=45°$



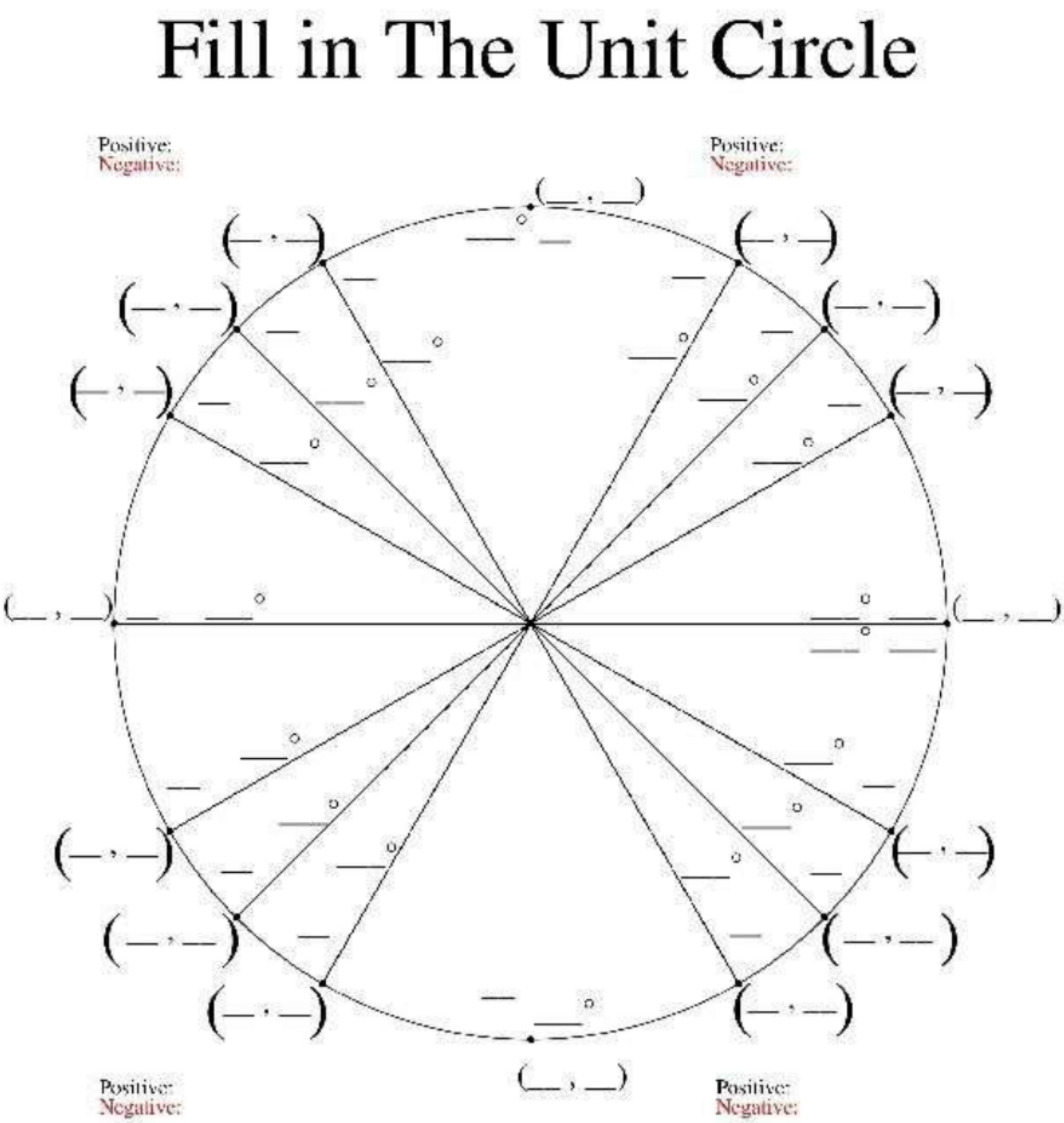
$\Theta=60°$



$\Theta=0°$ or $\Theta=90°$



Now let's do all four quadrants...



Coordinates (x, y) on the unit circle:

cosΘ = _____

sinΘ = _____

tanΘ = _____

13.4 The Sine Function (Day 1)

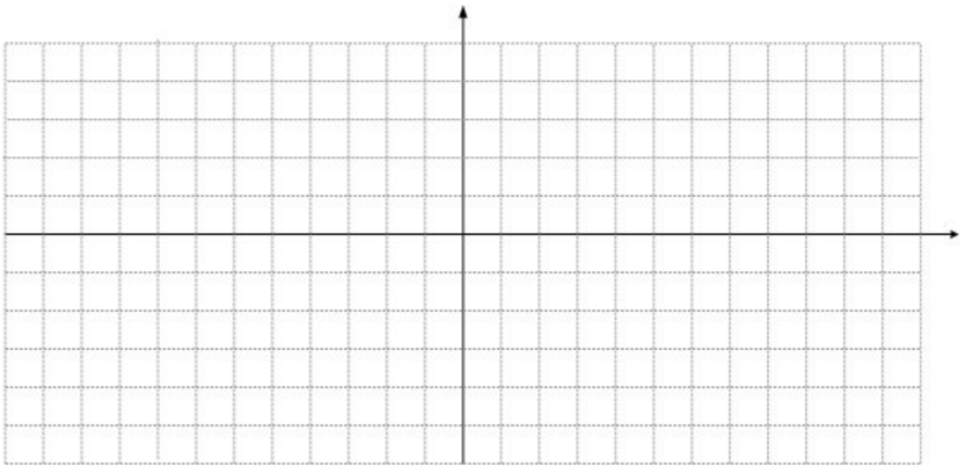
Warm Up: Use the graph. State:

1. the period
2. the domain
3. the amplitude
4. the range

sine function y=sinϴ: for each measure of ϴ, the sine of ϴ corresponds with the _____-coordinates on the unit circle.

y=sinϴ

ϴ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°
y								



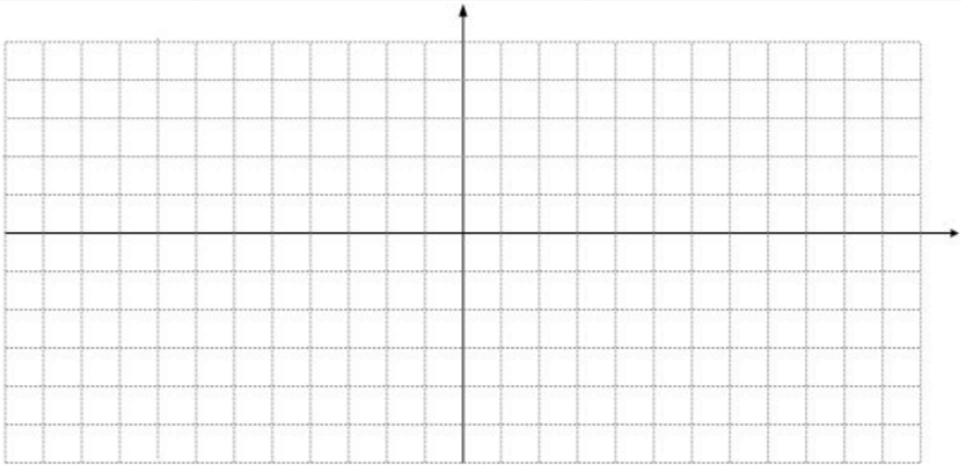
Example 1: Interpreting the Sine Function in Degrees

a. What is the value of y=sinϴ for ϴ=270°?
b. For what values of ϴ between 0° and 360° does the graph of y=sinϴ reach
   - the maxiumum value of y=1?
   - the minimum value of y=-1?
   - x-intercept of y=0? aka “zero”

Mathematical convention: An angle measure θ can be expressed in degrees or in radians. However, when no unit is mentioned, you should use radians. Let's graph the sine function in radians…

y=sinθ

θ (radians)					
y					



Example 2: Estimating Sine Values in Radians

Use your graph above to estimate the value. Check your estimate with a calculator.

a. sin 2

b. sin π

For the sine function, find the following:

a. amplitude

b. period (in degrees and radians)

c. domain and range

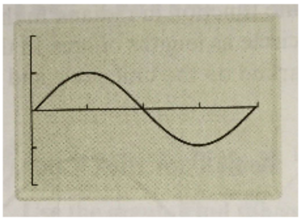
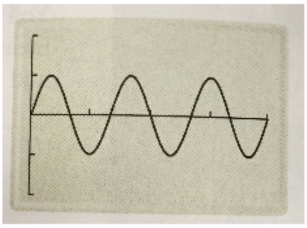
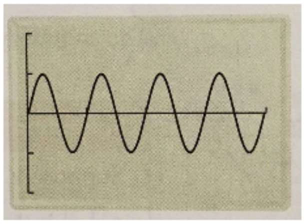
Properties of Sine Functions
Suppose **$y = a \sin b\theta$**, where $a \neq 0$, $b > 0$, and Θ in radians.

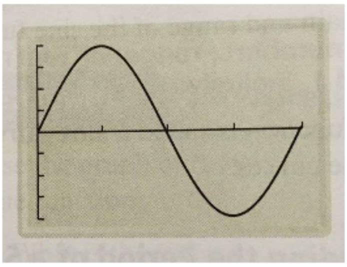
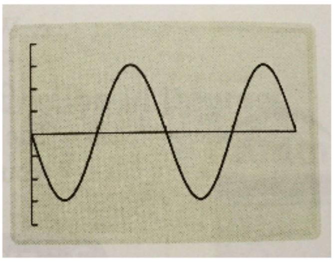
- The amplitude of the function is _____
- The number of cycles in the interval from 0 to 2π is _____
- The period of the function is _____

Examples 3&4: Finding the Period and Amplitude of a Sine Function

a. Find the amplitude.
b. How many cycles does the sine function have in the interval from 0 to 2π?
c. Find the period.

The Θ-axis represents values from 0 to 2π. Each interval on the y-axis represents 1 unit.

13.4 The Sine Function (Day 2)

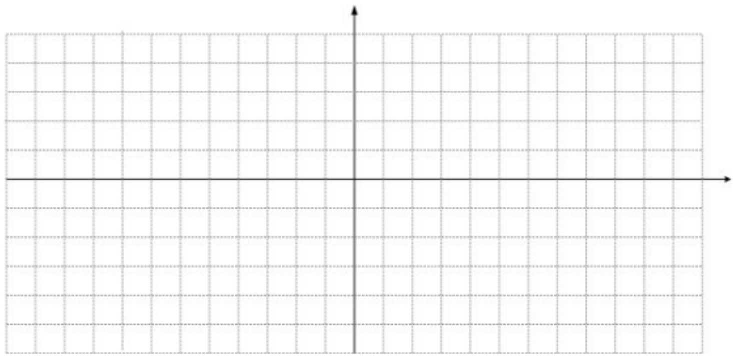
II. Graphing Sine Functions

You can use 5 points equally spaced through one cycle to sketch a sine curve. For a>0, this 5-point pattern is zero-max-zero-min-zero.

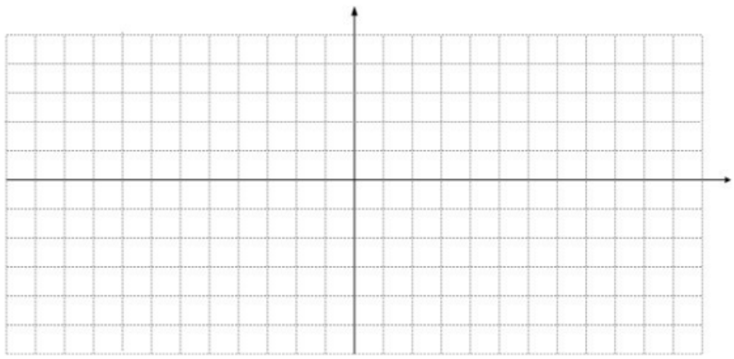
Example 5: Sketching a Graph

Sketch one cycle of each sine curve. Then write an equation for each graph.

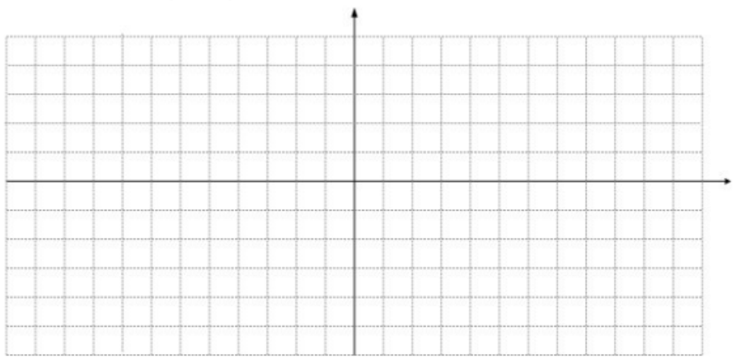
a. amplitude 2, period 4π, a>0



b. amplitude 3, period 4, a>0



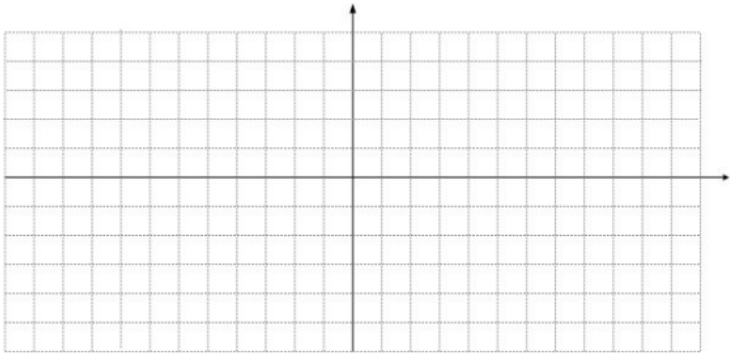
c. Predict the 5-point pattern for the sine function when a<0. Then sketch amplitude 2, period 2π/3.



Example 6: Graphing from a Function Rule

Sketch one cycle of the following sine functions.

1. $y = \frac{1}{2}sin2\theta$



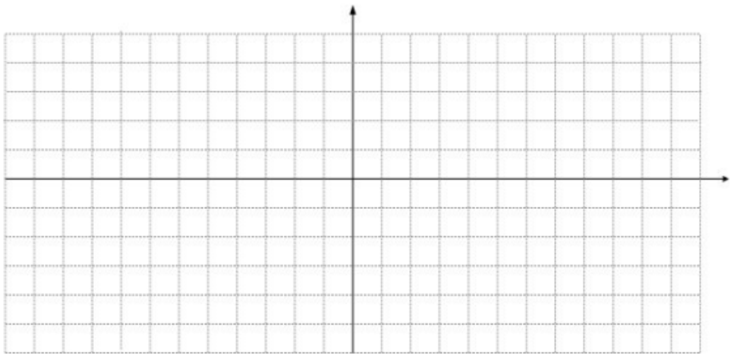
amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:

2. $y = 3sin\frac{\pi}{2}\theta$



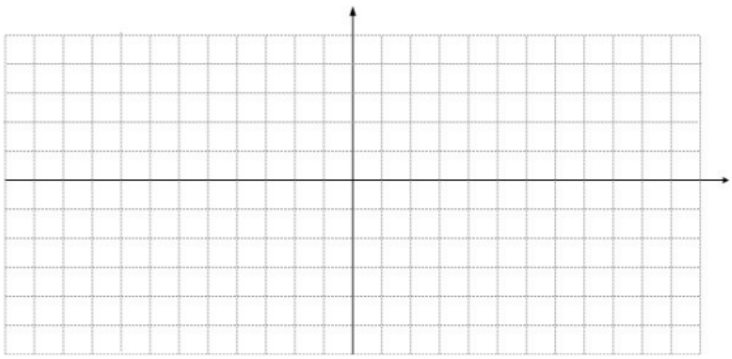
amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:

3. $y = -4sin\frac{1}{2}\theta$



amplitude:

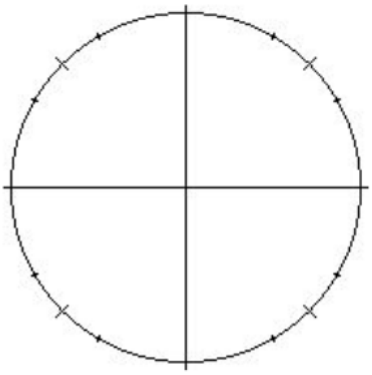
period:

interval spacing
on Θ-axis:

5-point pattern:

<u>13.5 The Cosine Function (Day 1)</u>

Warm up: Fill out the unit circle. Evaluate the following angles of cosine.

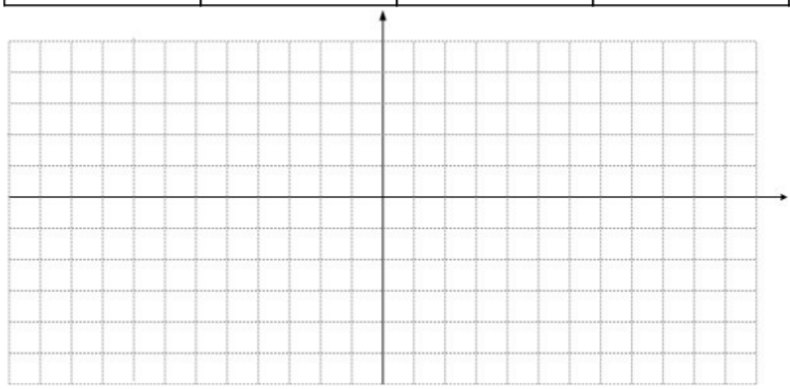


1. $\cos 0^\circ$
2. $\cos 90^\circ$
3. $\cos 180^\circ$
4. $\cos 270^\circ$
5. $\cos 360^\circ$
6. $\cos 0$
7. $\cos \frac{\pi}{2}$
8. $\cos \pi$
9. $\cos \frac{3\pi}{2}$
10. $\cos 2\pi$

Remember: cosΘ equals the _____-coordinate on the unit circle.

I. Graph the cosine function: y=cosΘ

Θ (radians)					
y					



Example 1: Interpreting the Graph of y=cosΘ

a. Use your graph above. Find the following:

domain period range amplitude

b. In the interval from 0 to 2π, where do the maximum occur? minimum? zeros?

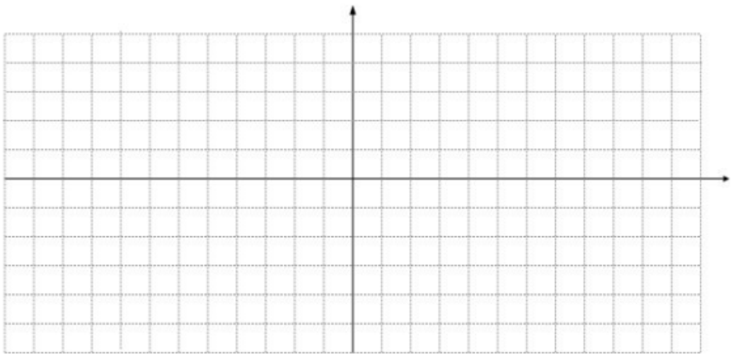
c. What is the 5-point pattern of the cosine graph?

Properties of Cosine Functions
Suppose **y = *a* cos *bθ***, where a≠0, b>0, and θ in radians.

- The amplitude of the function is _____
- The number of cycles in the interval from 0 to 2π is _____
- The period of the function is _____

Example 2: Sketching the Graph of a Cosine Function
Sketch one cycle of the following cosine functions.

1. $y = cos\frac{\pi}{2}\theta$



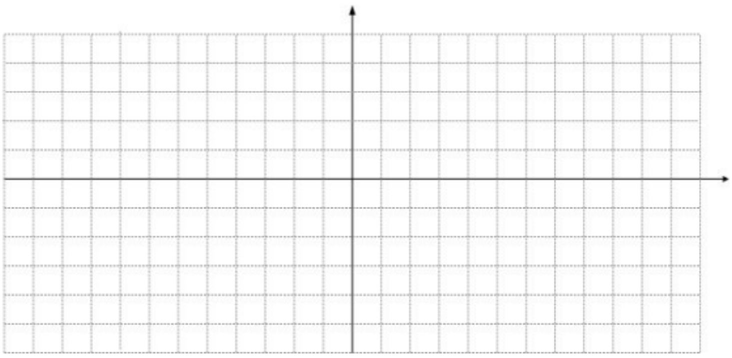
amplitude:

period:

interval spacing on θ-axis:

5-point pattern:

2. $y =- 3cos\theta$



amplitude:

period:

interval spacing on θ-axis:

5-point pattern:

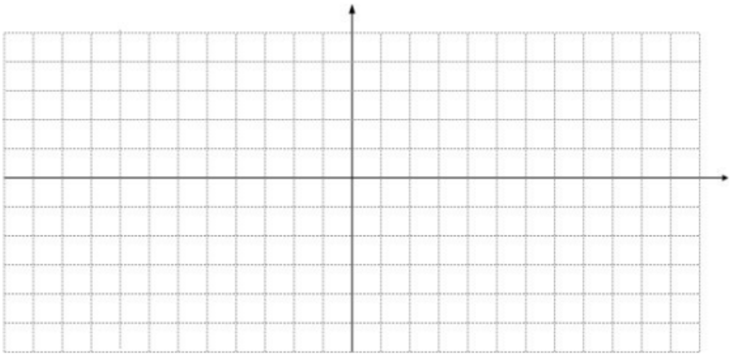
3. $y = 1.5cos2\theta$

amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:



13.5 The Cosine Function (Day 2)

Warm up: Graph the following cosine functions.

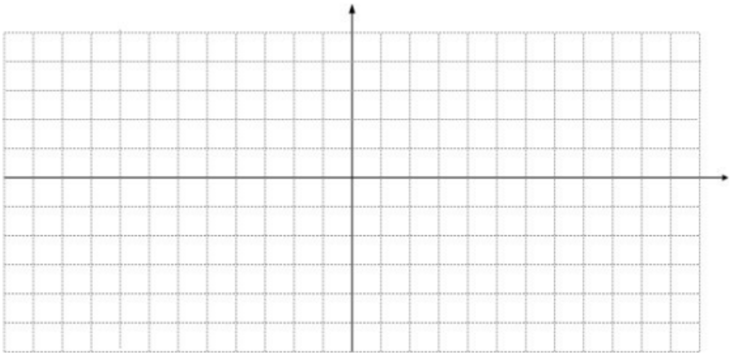
1. $y = 3cos2\theta$

amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:



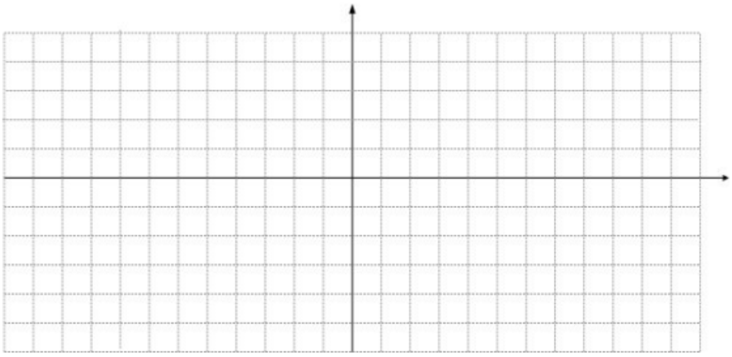
2. $y = -2cos\theta$

amplitude:

period:

interval spacing
on Θ-axis:

5-point pattern:



II. Solving Trigonometric Equations

Example 4: Solving a Cosine Equation

Solve the cosine equation in the interval from 0 to 2π. Round to the nearest hundredth. Calculators needed.

a. $-2cos\theta = 1.2$

b. $3cos2t = -2$

c. $5cos\frac{\pi}{2}t = 3$

Identify the period, range, and amplitude of each function.

22. $y = 3cos\theta$

24. $y = 2cos\frac{1}{2}t$

26. $y = 3cos\left(-\frac{\theta}{3}\right)$

28. $y = 16cos\frac{3\pi}{2}t$

13.6 The Tangent Function

Warm up: Use a calculator to find the sine and cosine of each Θ. Then calculate the ratio of sinΘ to cosΘ.

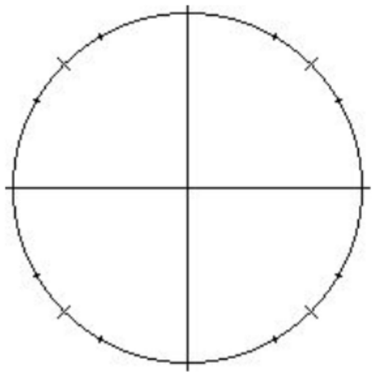
θ	$sin\theta$	$cos\theta$	$\frac{sin\theta}{cos\theta}$
1. $\frac{\pi}{3}$			
2. 30°			
3. 90°			
4. π			
5. $\frac{7\pi}{6}$			

I. The Tangent Function

The cosΘ is derived from the _____ - coordinate of the point on the unit circle.

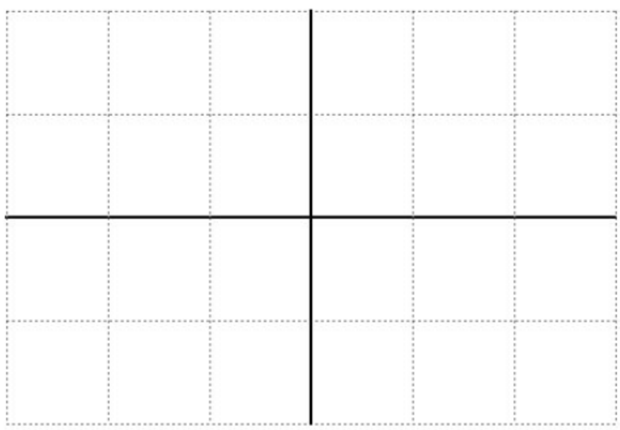
The sinΘ is derived from the _____ - coordinate of the point on the unit circle.

The tanΘ is derived from the ratio of sinΘ to cosΘ. In other words: tanΘ = _________



y=tanΘ

Θ (radians)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$
y							



Features of the parent tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

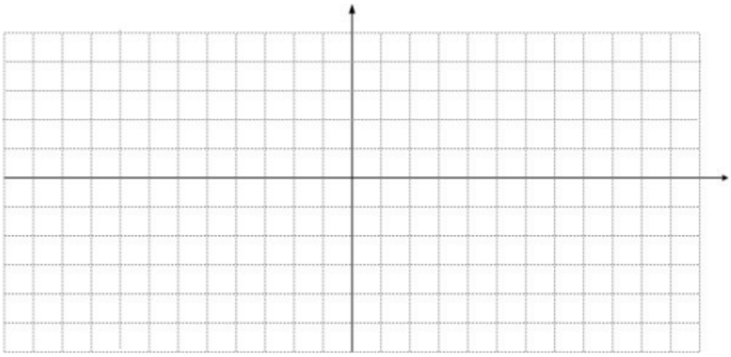
"guide points":

Properties of Tangent Functions
Suppose **y = *a* tan *bΘ***, where b>0, and Θ in radians.

- The period of the function is _____
- 1 cycle occurs in the interval from _________ to _________
- There are vertical aymptotes at each end of the cycle.
- The pattern is *"asymptote, -a, zero, a, asymptote"*.

Example 2: Graphing a Tangent Function
Sketch 2 cycles of each tangent function.

1. $y = tan\pi\theta$



period:

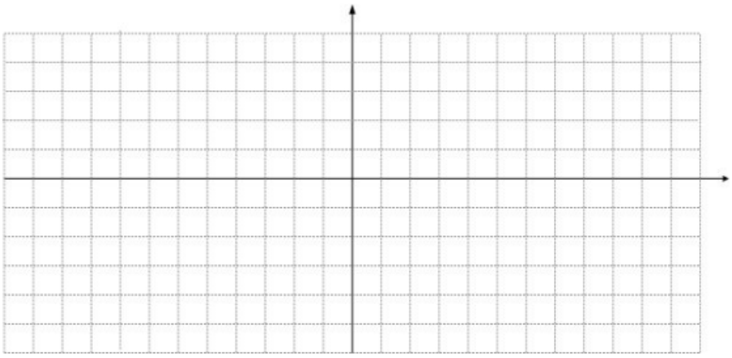
1 cycle: from ______ to ______

VA:

2 guide points:

pattern:

2. $y = tan3\theta$



period:

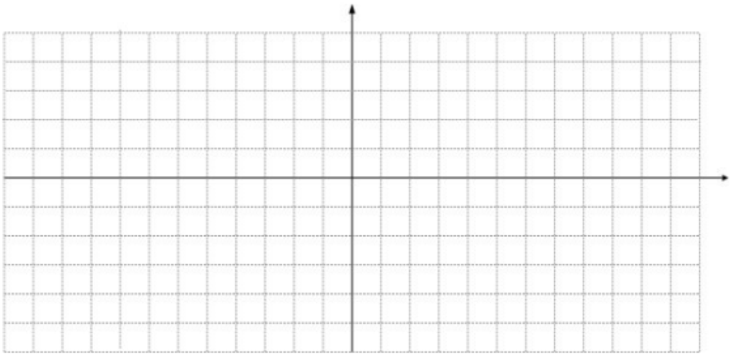
1 cycle: from ______ to ______

VA:

2 guide points:

pattern:

3. $y = tan\frac{\pi}{2}\theta$



period:

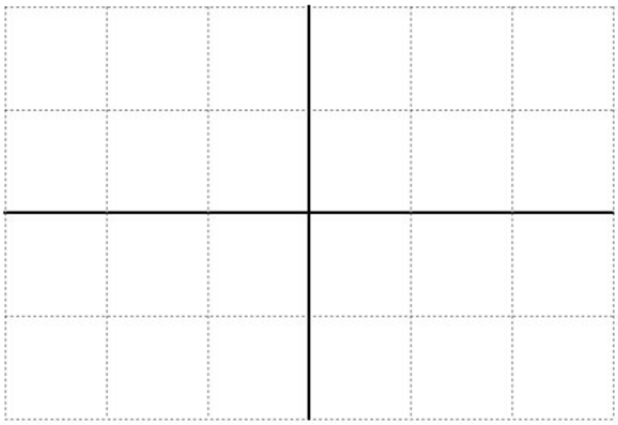
1 cycle: from ______ to ______

VA:

2 guide points:

pattern:

13.7 Translating Sine and Cosine Functions
Warm Up:
1. Graph y=tanΘ … again! (Try not to peek at prior notes.)



Features of the tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

"guide points":

2. Compare each pair of equations. State the translations (horizontal, vertical) involved.

a. $y = 2x,\ \ y = 2x + 5$ b. $y = |x|,\ \ y = |x + 3|$ c. $y = x^2,\ \ y = x^2 - 4$

d. $y = |x - 2| + 1$ e. $y = x^2,\ \ y = (x + 3)^2 - 6$ d. $y = f(x),\ \ y = f(x - h) + k$

I. Graphing Translations of Trigonometric Functions
Phase shift - the horizontal translation of a function. If *f(x)* is the "parent", then *f(x-h)* translates horizontally *h* units. For example: *f(x-1)* translates _______________, *f(x+3)* translates _______________.
Vertical shift - the vertical translation of a function. If *f(x)* is the "parent", then *f(x)+k* translates vertically k units. For example: *f(x) - 1* translates _______________, *f(x) + 3* translates _______________.

Example 1: Identifying Phase Shifts and Vertical Shifts
What is the value of *h* and *k* in each translation? Describe the shift i.e. "3 units to the left".

a. f(x-2) b. y= cos(x+4) c. f(t-5) d. y= sin(x+3)

e. f(x) - 2 b. y= cosx + 4 c. f(t) - 5 d. y= sinx + 3

Example 2: Graphing Translations

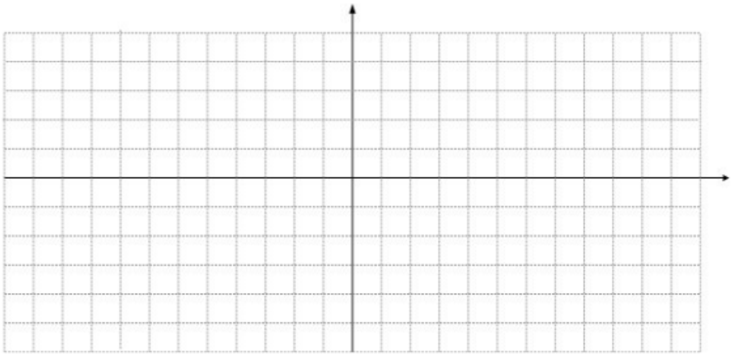
Make a table and then graph the following functions on the same set of axes:

y= sinx

x					
y					

y= sinx + 3

x					
y					

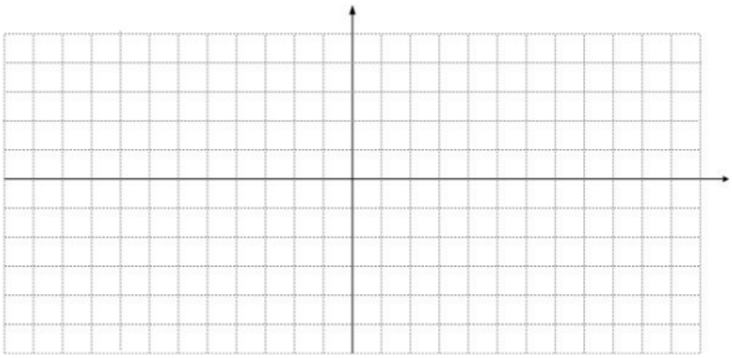


$y = cosx$

x					
y					

$y = cos(x - \frac{\pi}{2})$

x					
y					



Example 3: Graphing a Combined Translation

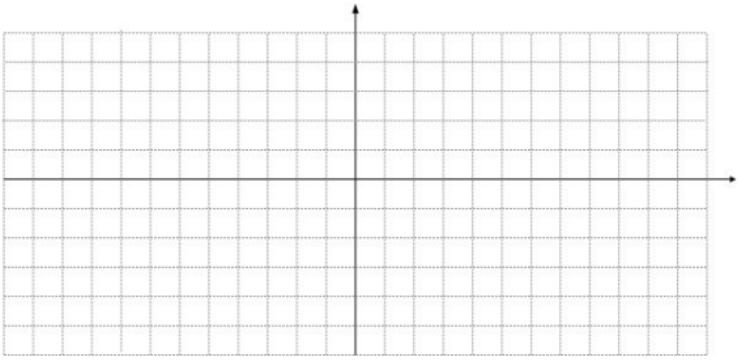
1. $y = sin(x + \pi) - 2$

amplitude:

period:

phase shift:

interval spacing on x-axis:



vertical shift: 5 point pattern:

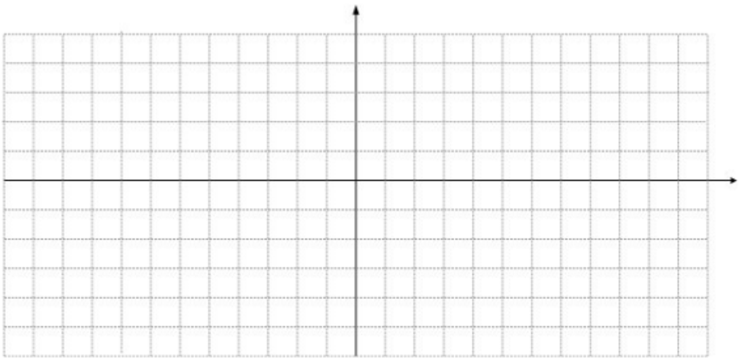
2. $y = 2cos(x - \frac{\pi}{2}) + 3$

amplitude:

period:

phase shift:

interval spacing on x-axis:



vertical shift: 5 point pattern:

Summary: Families of Sine and Cosine Functions

Parent	**Transformed Function**
y=sinx	____________________
y=cosx	____________________

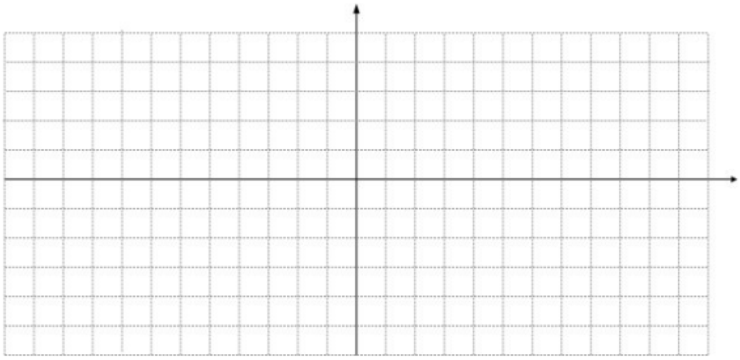
amplitude = h =

period = k =

13.7 Translating Sine and Cosine Functions (Day 2)
Warm Up:

$y = -3sin(x + \frac{\pi}{2}) + 2$

amplitude:



period:

phase shift:

interval spacing on x-axis:

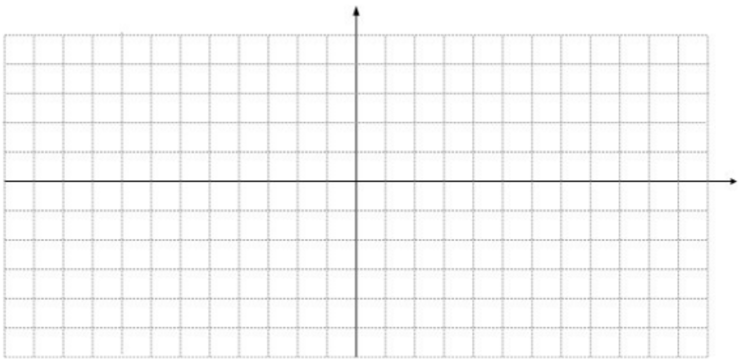
vertical shift:

5 point pattern:

When the phase shift is a pesky number...

$y = sin(x - \frac{\pi}{3})$

amplitude:



period:

phase shift:

interval spacing on x-axis:

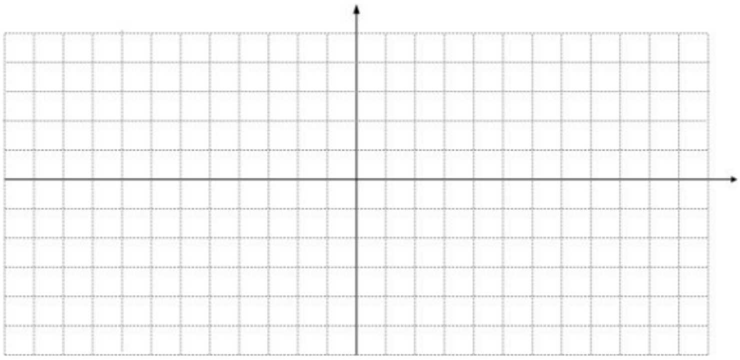
vertical shift:

5 point pattern:

Graphing a translation of y = sin 2x...

$y = sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

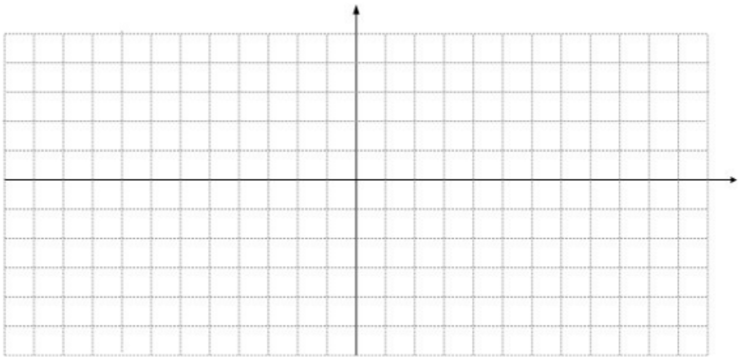
interval spacing on x-axis:

vertical shift:

5 point pattern:

$y =- 3sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:

$y = 2cos\frac{\pi}{2}(x + 1) - 3$

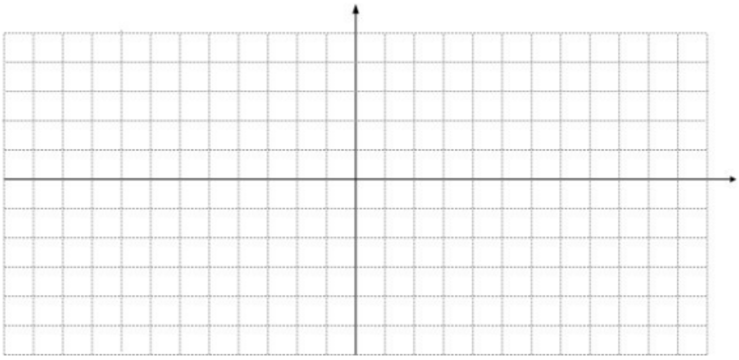
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

Example 5: Writing a Translation

Write an equation for each translation.

a. $y = sin\pi,\ \ \pi\ units\ down$

b. $y = -\ cosx,\ \ 2\ units\ left$

c. $y = cosx,\ \ \frac{\pi}{2} units\ up$

d. $y = 2sinx,\ \ \frac{\pi}{4} units\ right$

13.8 Reciprocal Trigonometric Functions

Warm up:

Find the reciprocal of each fraction:

1. $\frac{9}{13}$ 2. $-\frac{5}{8}$ 3. $\frac{1}{2\pi}$ 4. $\frac{14}{-t}$ 5. θ

Name the 3 trigonometric functions you have studied so far:

1. ___________
2. ___________
3. ___________

These 3 trigonometric functions have reciprocals.

Definition: Cosecant, Secant, and Cotangent

Example 1: Using Reciprocals

a. Use your calculator (degree mode). Round your answer to the nearest hundredth.

csc 60° cot 55° sec 15°

b. Suppose $cos\theta = \frac{5}{13}$. Find $sec\theta$.

c. Suppose $sin\theta = \frac{-12}{13}$. Find $csc\theta$.

Example 2: Find The Exact Value

csc 30° csc 45° csc 60° csc 90°

sec 30° sec 45° sec 60° sec 90°

cot 30° cot 45° cot 60° cot 90°

Example 3: Using Radians

a. Use your calculator (radian mode). Round your answer to the nearest hundredth.

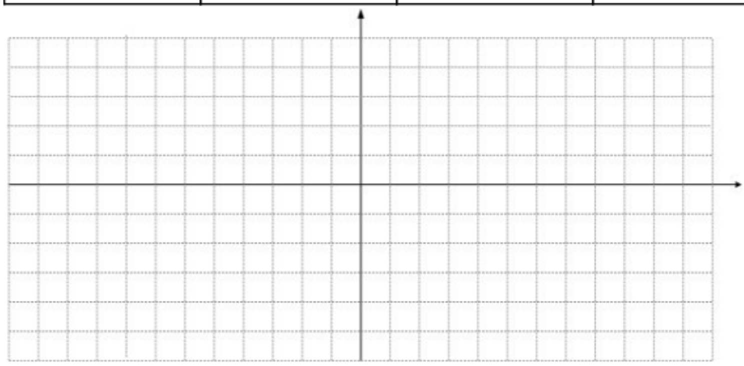
$sec(-1)$ $\qquad$ $csc(-1.5)$ $\qquad$ $sec2$

b. Find the exact value.

$cot\frac{\pi}{3}$ $\qquad$ $cot\pi$ $\qquad$ $sec0$

Example 4: Graph The Reciprocal Trigonometric Functions

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= sin x					
y= csc x					



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= cos x					
y= sec x