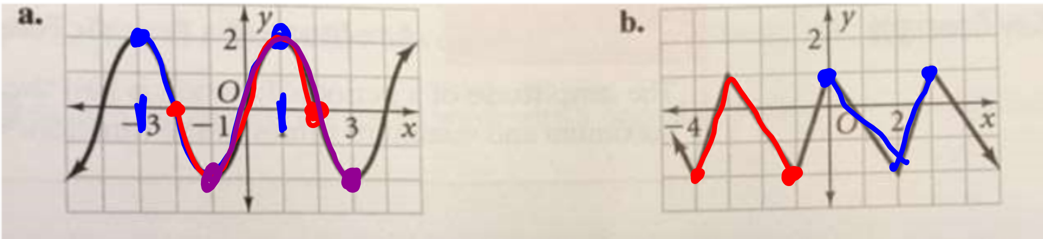
13.1 Exploring Periodic Data

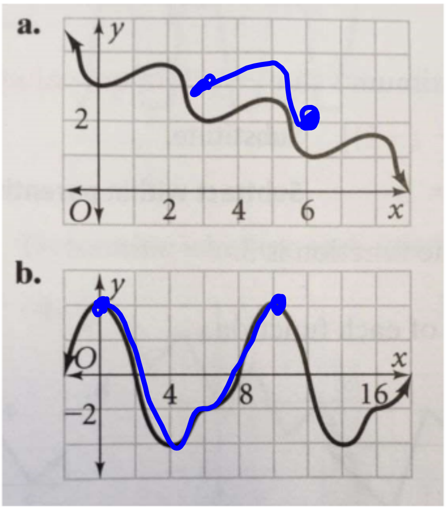
Periodic function - repeats a pattern of y-vales (outputs) at regular intervals.
Cycle - 1 complete pattern. A cycle may begin at any point on the graph of the function.
Period - the horizontal length of 1 cycle, - in terms of x-values.

Example 1: Identifying Cycles and Periods
Analyze the periodic function. a) Identify 1 cycle in two different ways. b) Determine the period of the function.



$1-(-3)=4$ units $\qquad$ $3-0=3$ units

Example 2: Identifying Periodic Functions
Determine whether each function is or is not periodic. If it is, find the period.



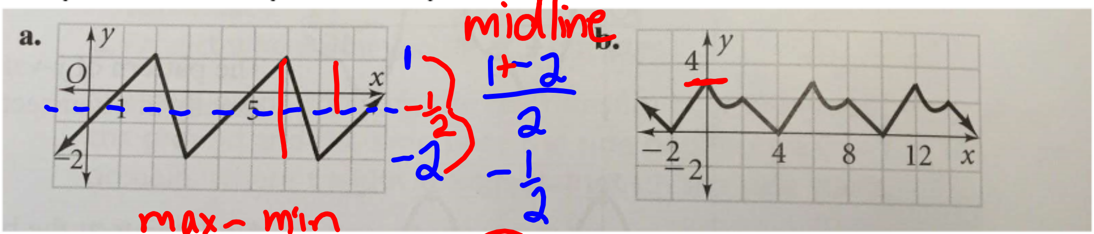
not periodic

periodic
9 units

$9-0$

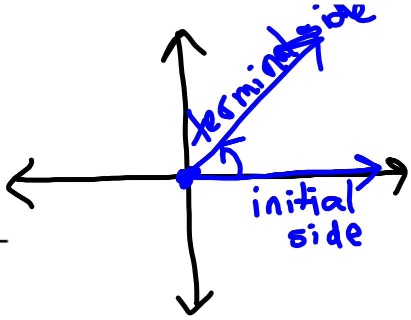
Amplitude - of a periodic function measures the amount of variation in the y-values. To find the amplitude:

Example 3: Find the Amplitude of the periodic function.



amp: $\frac{1-(-2)}{2} = \frac{3}{2}$

amp: $\frac{max_y - min_y}{2} = \frac{3-0}{2} = \frac{3}{2}$



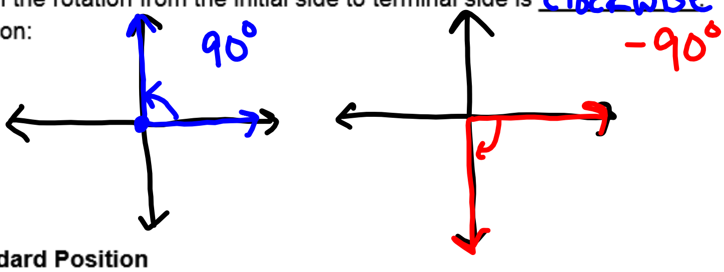
13.2 Angles (day 1)

An **angle in standard position** has:

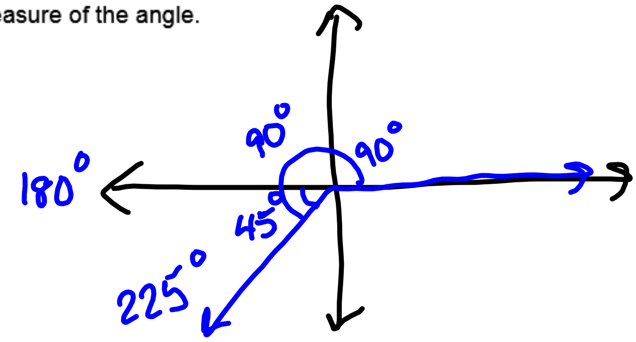
- vertex is at the origin
- one ray is on the x-axis

Initial side - the ray on the positive x-axis
Terminal side - the other ray of the angle
The **measure of an angle in standard position** is the amount of rotation from the initial side to the terminal side.
The measure of an angle is **positive** when the rotation from the initial side to terminal side is counterclockwise
The measure of an angle is **negative** when the rotation from the initial side to terminal side is clockwise

Sketch and angle in standard position:



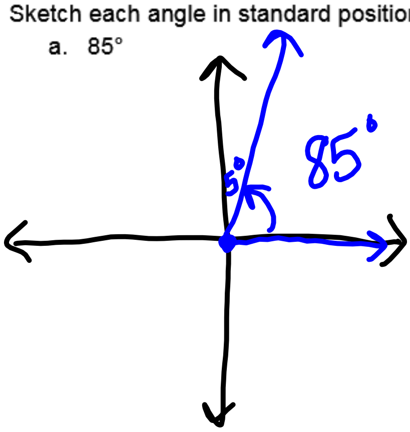
Example 1: Measuring an Angle in Standard Position
Find the measure of the angle.



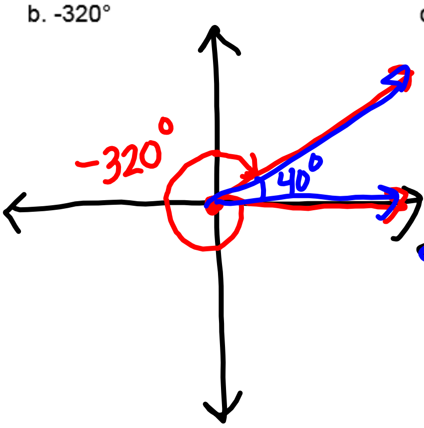
$90+90+45$

Example 2: Sketching an Angle in Standard Position
Sketch each angle in standard position.

a. 85°



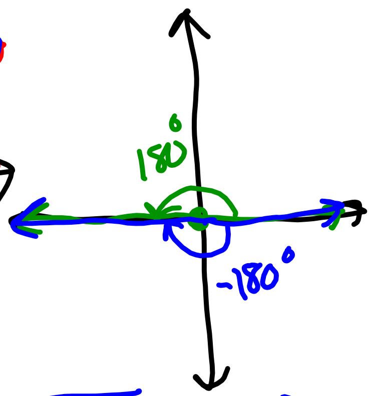
b. -320°



$\boxed{-320}+360$

$\boxed{40^\circ}$

c. 180°



$\boxed{180^\circ}-360^\circ$

$\boxed{-180^\circ}$

Coterminal Angles - Two angles in standard position are coterminal if they have the same terminal side. For any given angle, there are infinitely many coterminal angles. To find a coterminal angle, **add or subtract 360°.**

Example 3: Finding Coterminal Angles

a. Find a positive angle and a negative angle that are coterminal with 198°.

$198^\circ + 360^\circ = 558^\circ$

$198^\circ - 360^\circ = -162^\circ$

b. Are the angles with measure 40° and 680° coterminal? Explain.

40° 680°. not coterminal. the difference is not a multiple of 360°

$680 - 40 = \frac{640}{360} = \times$

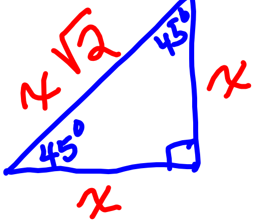
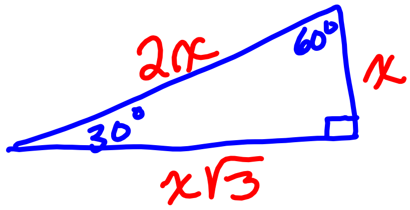
c. Find the measure of an angle between 0° and 360° coterminal with 385°.

$385^\circ - 360^\circ = 25^\circ$

d. Find the measure of an angle between 0° and 360° coterminal with -356°.

$-356^\circ + 360 = 4^\circ$

Recall: Special Right Triangles

45°-45°-90° Let a leg be x.	30°-60°-90° Let the shorter leg be x.
	

Find the missing side lengths in each 45°-45°-90° triangle. Rationalize any denominators.

1. hypotenuse = 1 inch

$x\sqrt{2} = 1$

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{2}}{2} x}$ $\boxed{x = \frac{\sqrt{2}}{2}}$

2. leg = 2 cm

$\boxed{\frac{x\sqrt{2}}{2\sqrt{2}}}$ $\boxed{x = 2}$

$2 = x$

3. hypotenuse = √3 ft

$\sqrt{3} = x\sqrt{2}$

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

$x = \frac{\sqrt{3}\sqrt{2}}{\sqrt{2}\sqrt{2}}$

$\boxed{x = \frac{\sqrt{6}}{2}}$

Find the missing side lengths in each 30°-60°-90° triangle. Rationalize any denominators.

4. shorter leg = 3 inch

$3 = x$

$2x = \boxed{6in}$

$x\sqrt{3} = \boxed{3\sqrt{3}in}$

5. longer leg = 1 cm

s: $x = \boxed{\frac{\sqrt{3}}{3} cm}$

l: $x\sqrt{3} = 1$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$x = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{\sqrt{3}}{3}$

h: $2x = \boxed{\frac{2\sqrt{3}}{3} cm}$

$2 \cdot \frac{\sqrt{3}}{3} =$

6. hypotenuse = 1 ft

s: $x = \boxed{\frac{1}{2} ft}$

l: $x\sqrt{3} = \frac{1}{2} \cdot \sqrt{3}$ $\boxed{\frac{\sqrt{3}}{2} ft}$

h: $\frac{2x}{2} = \frac{1}{2}$

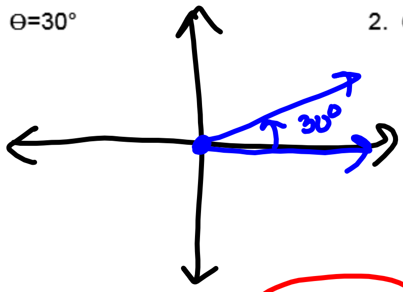
$x = \frac{1}{2}$

θ angle

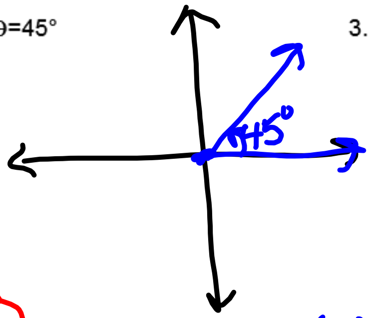
13.2 The Unit Circle (day 2)

Warm Up: Sketch the angle in standard position:

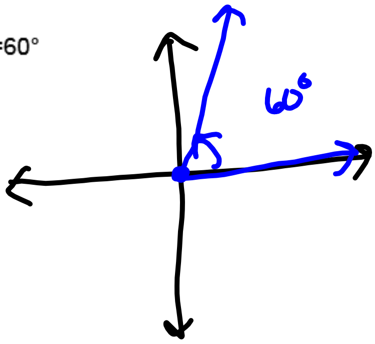
1. θ=30°



2. θ=45°



3. θ=60°



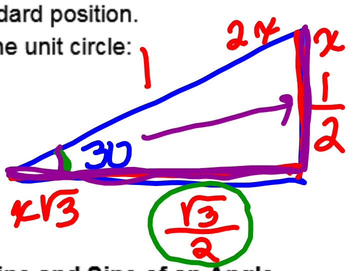
Unit Circle - a circle with a radius of __1__ and its center is at the __origin__

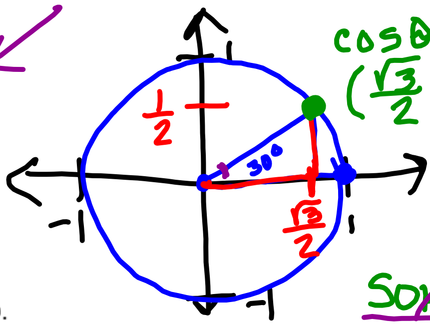
Points on the unit circle are related to periodic functions. You can use the symbol θ "theta" for the measure of an angle in standard position.

Sketch the unit circle:

$\cos 30°$

$\frac{adj}{hyp} = \frac{\frac{\sqrt{3}}{2}}{1}$





$\cos\theta$ $\sin\theta$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin 30°$

$\frac{opp}{hyp} = \frac{\frac{1}{2}}{1}$

SOH CAH TOA

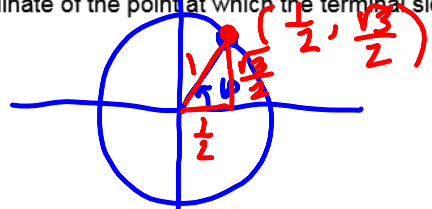
Definition: Cosine and Sine of an Angle

Given: an angle in standard position with measure θ.

The **cosine of θ** is the __x__-coordinate of the point at which the terminal side intersects the unit circle.

The **sine of θ** is the __y__-coordinate of the point at which the terminal side intersects the unit circle.

Sketch:

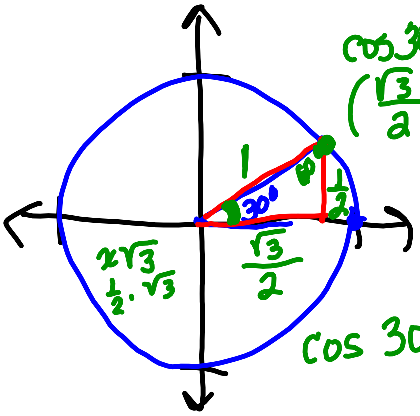


$\cos 60° = \frac{1}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$

Example 4&5: Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

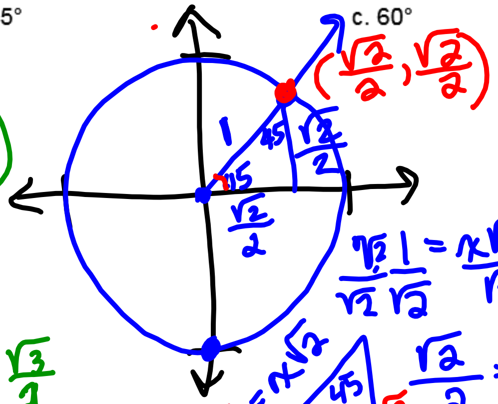
a. 30°



$\cos 30° = \frac{\sqrt{3}}{2}$

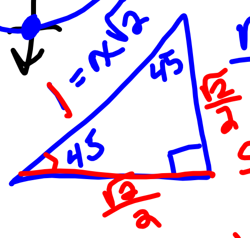
$\sin 30° = \frac{1}{2}$

b. 45°



c. 60°

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$



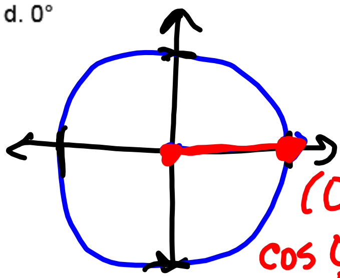
$\frac{\sqrt{2}}{2} = x$

$\sin = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}/1 = \frac{\sqrt{2}}{2}$

Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

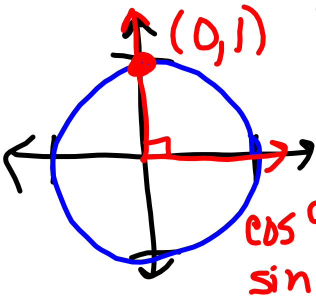
d. 0°



$\cos 0° = 0$

$\sin 0° = 0$

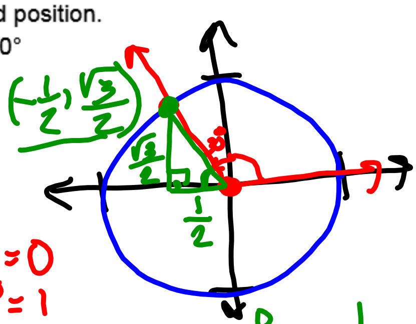
e. 90°



$\cos 90° = 0$

$\sin 90° = 1$

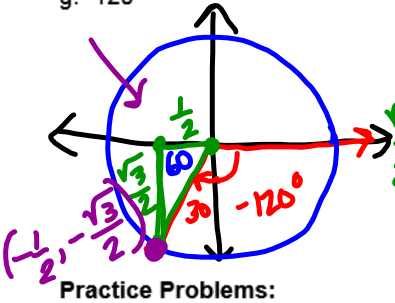
f. 120°



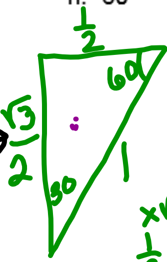
$\cos 120° = -\frac{1}{2}$

$\sin 120° = \frac{\sqrt{3}}{2}$

g. -120°



h. -60°



$\sin 60° = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{adj}{hyp} = \frac{1}{2}$

$\times\sqrt{3}$

$\frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$

$\sin(-120°) = -\frac{\sqrt{3}}{2}$

$\cos(-120°) = -\frac{1}{2}$

i. 150°

Practice Problems:

Calculator Needed: For angles that are not a multiple of 30° or 45°, you will need your calculator. Find cosΘ and sinΘ.

1. Θ=32°

$\cos 32° = 0.834$

$\sin 32° = 0.551$

2. Θ=-210°

$\cos(-210)° = -0.884$

$\sin(-210)° = -0.468$

3. Θ=-10°

Find a positive and negative coterminal angle for the given angle. $\pm 360°$

4. 3. Θ=400°

$(+360) = 760°$

$400 - 360 = 40°$

$40 - 360 = -320°$

5. Θ=-125°

$-125 + 360 = 235°$

$-125 - 360 = -485°$

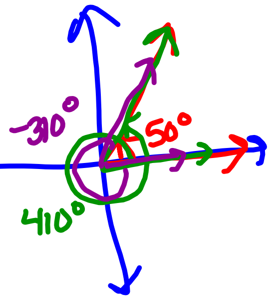
6. 3. Θ=-57°

$\boxed{-417°}$

$-57 - 360$

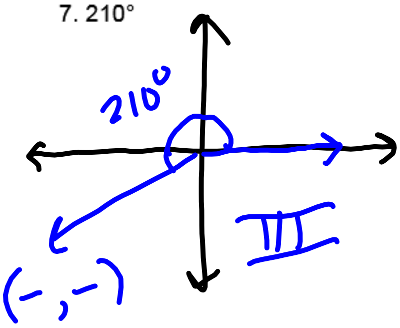
$-57 + 360$

$\boxed{303°}$

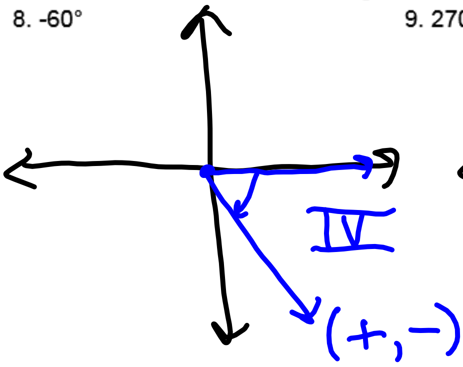


In which quadrant, or on which axis, does the terminal side of each angle lie? Sketch the angle to help you.

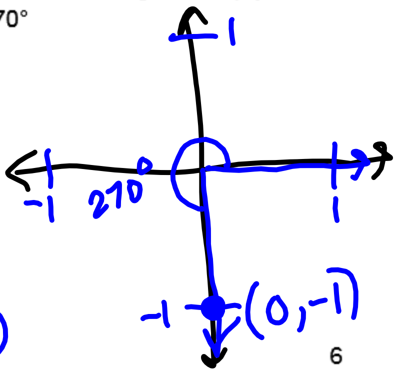
7. 210°



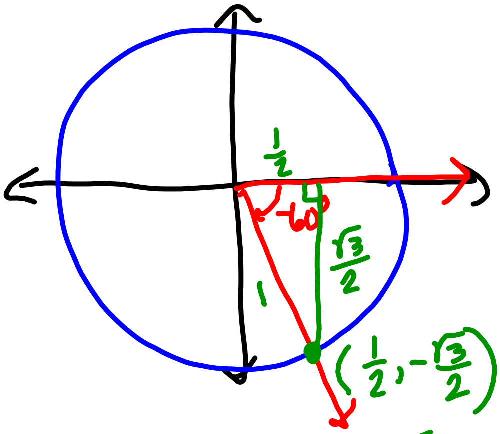
8. -60°



9. 270°



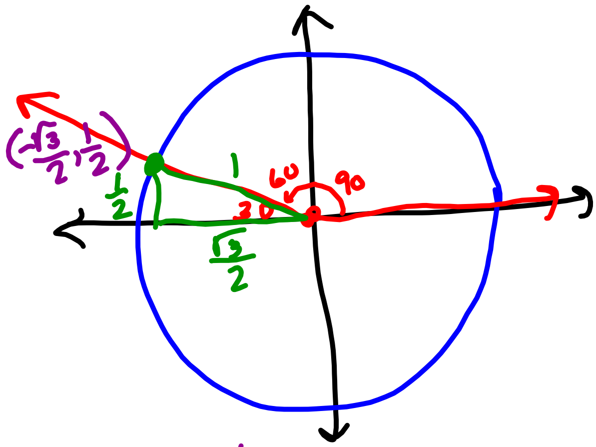
h. $-60°$



$\sin(-60°) = -\frac{\sqrt{3}}{2}$

$\cos(-60°) = \frac{1}{2}$

i. $150°$



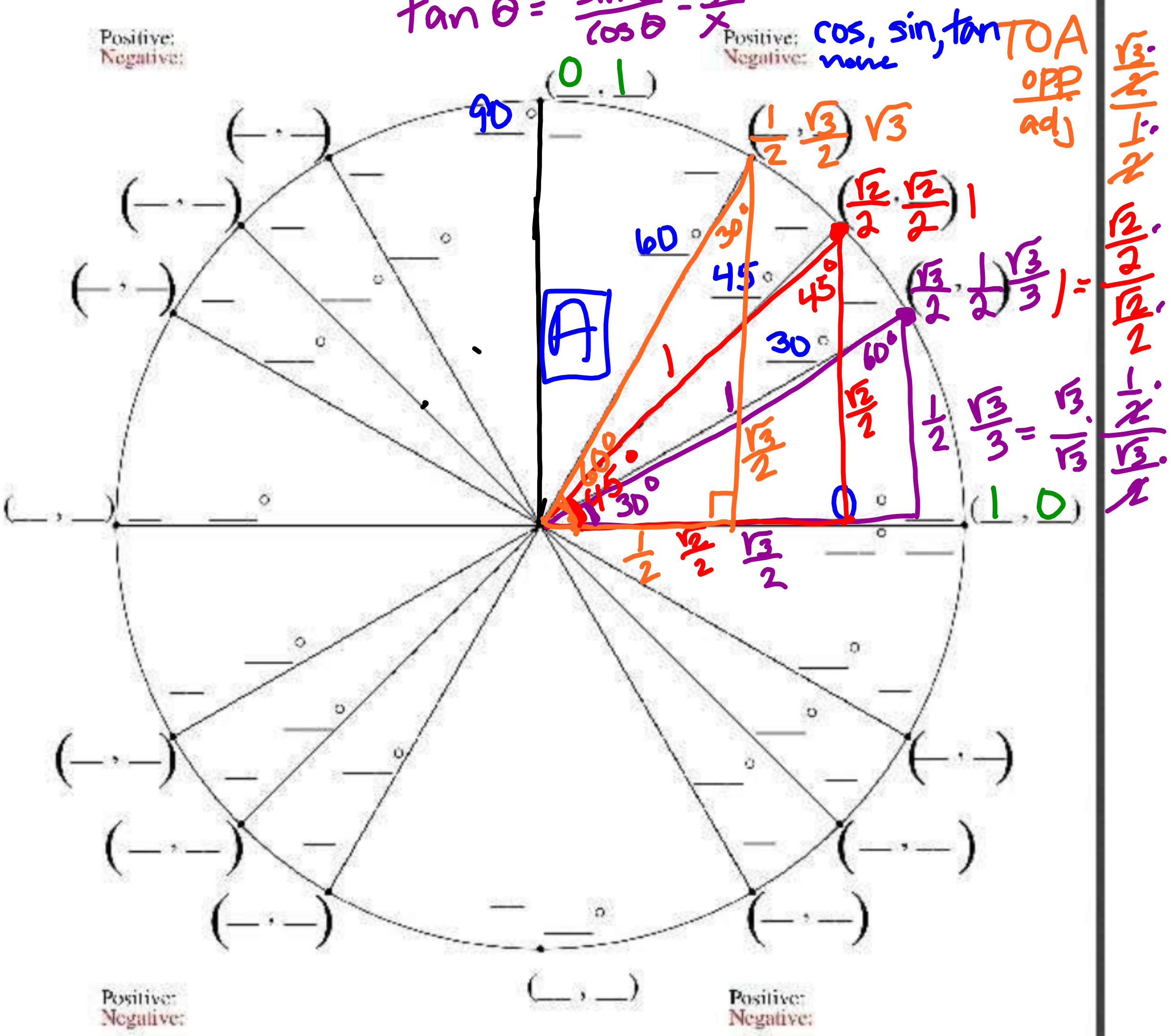
y: $\sin 150° = \frac{1}{2}$

x: $\cos 150° = -\frac{\sqrt{3}}{2}$

(cos θ, sin θ)

Fill in The Unit Circle

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$



13.3 Radian Measure (Day 1)

Warm Up: Find the circumference of a circle with the given radius or diameter. Round your answer to the nearest tenth.

1. radius 4 in $C = 2\pi r$

$C = 2\pi \cdot 4 = 8\pi$

25.1 in

2. diameter 70 m $C = d\pi = 2\pi r$

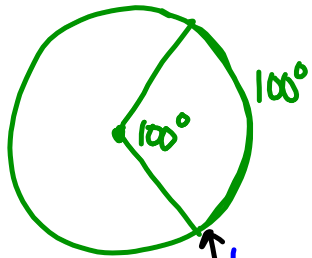
$C = 70\pi$

219.9 m

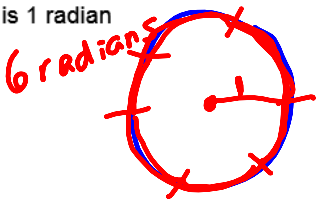
Central angle - an angle whose vertex is the center of a circle

Intercepted arc - the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

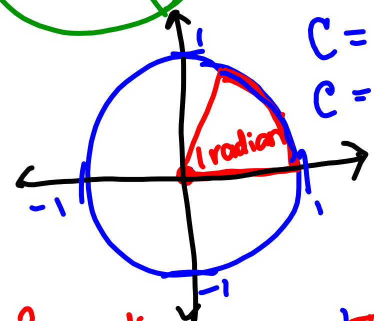
sketch:



Radian - when the intercepted arc equals the radius, the measure of the angle is 1 radian



sketch:

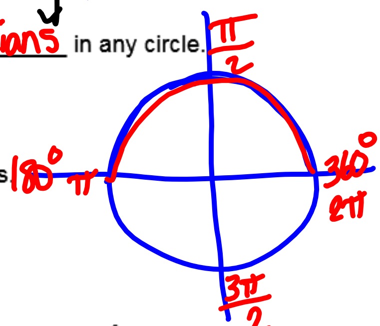


radians

$C = 2\pi \cdot 1$

$C = 2\pi$

- The circumference of a circle is $2\pi r$. Thus there are 2π radians in any circle.
- Since $\frac{2\pi}{2}$ radians = $\frac{360}{2}$°, then π radians = 180 °
- Thus you can use this proportion to convert between degrees and radians.



$D \to R: \frac{\pi}{180°}$ $R \to D: \frac{180°}{\pi}$

Example 1: Use a proportion

a. Find the radian measure of 60°.

$\cancel{60}^{\circ} \cdot \frac{\pi}{\cancel{180}^{\circ}_{3}} = \frac{\pi}{3}$

b. Find the degree measure of $\frac{5\pi}{2}$ *radians*

$\frac{5\cancel{\pi}}{\cancel{2}} \cdot \frac{\overset{90}{\cancel{180}}^{\circ}}{\cancel{\pi}} = 450°$

Converting between Radians and Degrees

- To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$
- To convert radians to degress, multiply by $\frac{180^\circ}{\pi}$

Example 2: Using Dimensional Analysis

Convert the angle to degrees. Round to the nearest degree.

a. $-\frac{3\pi}{4}$ *radians*

$-\frac{3\cancel{\pi}}{\cancel{4}} \cdot \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}}$

-135°

b. $\frac{\pi}{2}$ *radians*

$\frac{\pi}{2} \cdot \frac{180^\circ}{\pi}$

90°

c. 2 *radians*

$2 \text{ radians} \cdot \frac{180^\circ}{\pi \text{ rad.}}$

$\frac{360^\circ}{\pi}$

Convert the angle to radians. Round to the nearest hundredth.

d. 27°

$\overset{3}{\cancel{27}}^\circ \cdot \frac{\pi}{\underset{20}{\cancel{180^\circ}}}$

$\boxed{\frac{3\pi}{20}}$ radians

e. 225°

$\overset{5}{\cancel{225}}^\circ \cdot \frac{\pi}{\underset{4}{\cancel{180^\circ}}}$

$\frac{5\pi}{4}$ radians

f. 150°

$\overset{5}{\cancel{150}}^\circ \cdot \frac{\pi}{\underset{6}{\cancel{180^\circ}}}$

$\boxed{\frac{5\pi}{6} \text{ radians}}$

Example 3: Find the exact values of cosϴ and sinϴ for each angle measure.

Step 1: Convert to degrees.

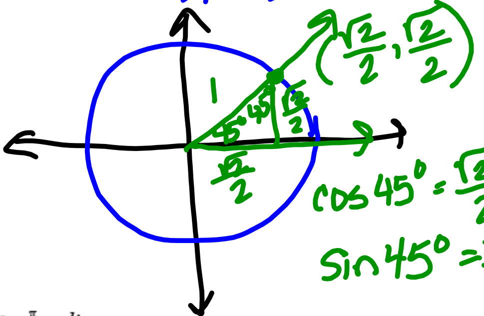
Step 2: Draw the angle. The terminal side is the hypotenuse.

Step 3: Complete the right triangle. Draw a leg to the ***x-axis***.

Step 4: State the cosϴ and sinϴ.

a. $\frac{\pi}{4}$ *radians*

$\frac{\cancel{\pi}}{4} \cdot \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}} = 45^\circ \rightarrow \frac{\pi}{4}$

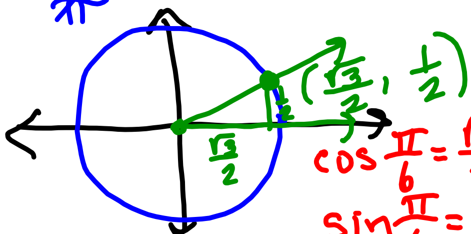


$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

b. $\frac{\pi}{6}$ *radians*

$\frac{\cancel{\pi}}{\cancel{6}} \cdot \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}} = 30^\circ \rightarrow \frac{\pi}{6}$

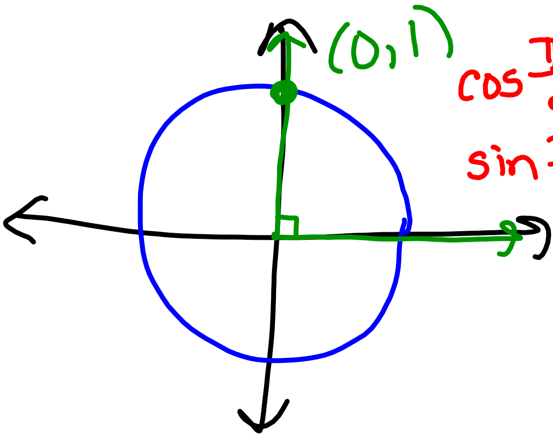


$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{6} = \frac{1}{2}$

c. $\frac{\pi}{2}$ *radians*

$\frac{\cancel{\pi}}{\cancel{2}} \cdot \frac{\overset{90}{\cancel{180}}}{\cancel{\pi}} = 90^\circ \rightarrow \frac{\pi}{2}$

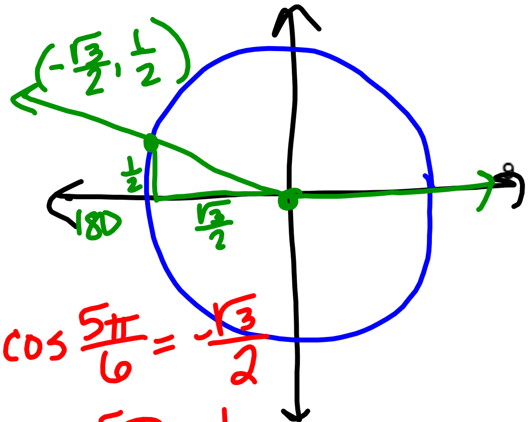


$\cos \frac{\pi}{2} = 0$

$\sin \frac{\pi}{1} = 1$

d. $\frac{5\pi}{6}$ *radians*

$\frac{5\pi}{\cancel{6}} \cdot \frac{\overset{30}{\cancel{180}}}{\pi} = 150^\circ$



$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

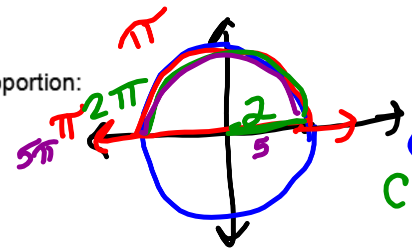
$\sin \frac{5\pi}{6} = \frac{1}{2}$

13.3 Arclength (Day 2)

Warm Up: Convert the angle to degrees. Draw the angle in standard position, as well as the corresponding right triangle. Then find the exact values for cosΘ and sinΘ.

$\theta = \frac{\pi}{3}$

You can find the length of an intercepted arc by using the proportion:



Unit Circle $r = 1$

$s = \theta$

$C = 2\pi r$

$C = 2\pi$

$C = 2 \cdot \pi \cdot 2$

$= 4\pi$

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure Θ (in *radians*), the length *s* of the intercepted arc is:

$$\boxed{s = \theta r}$$

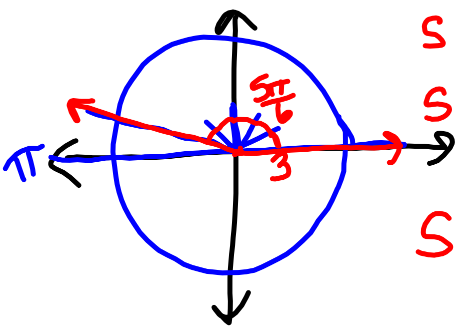
$C = 2 \cdot \pi \cdot 5$

$= 10\pi$

Example 4: Finding the Length of an Arc

Find the length of the intercepted arc to the nearest tenth. Sketch a diagram!

a. Given: A circle of radius 3 in, $\theta = \frac{5\pi}{6}$.

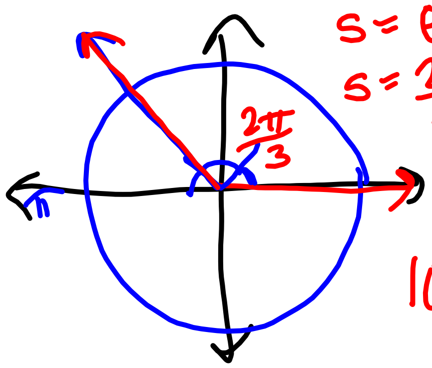


$s = \theta r$

$s = \frac{5\pi}{6}(3)$

$s = 7.9$ in

b. Given: A circle of radius 5m, $\theta = \frac{2\pi}{3}$.



$s = \theta \cdot r$

$s = \frac{2\pi}{3} \cdot 5$

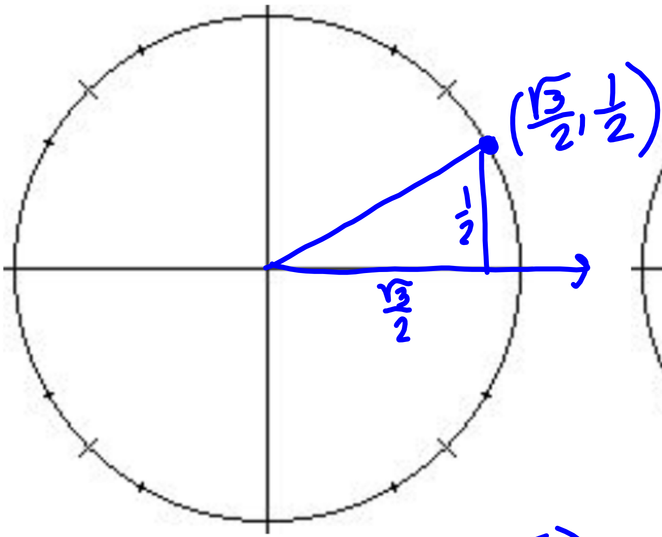
$\frac{10\pi}{3}$ m

10.5m

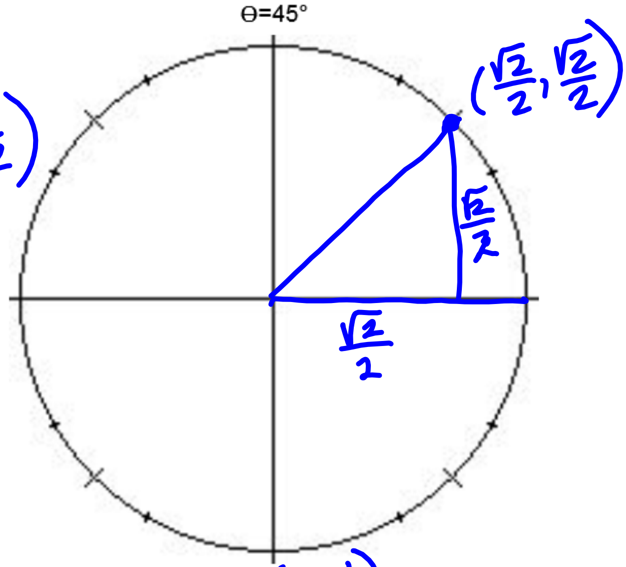
The Unit Circle: radius = 1

If you know quadrant 1, you can derive quadrants 2, 3, 4 by symmetry. Thus, let's study quadrant 1.

Θ=30°

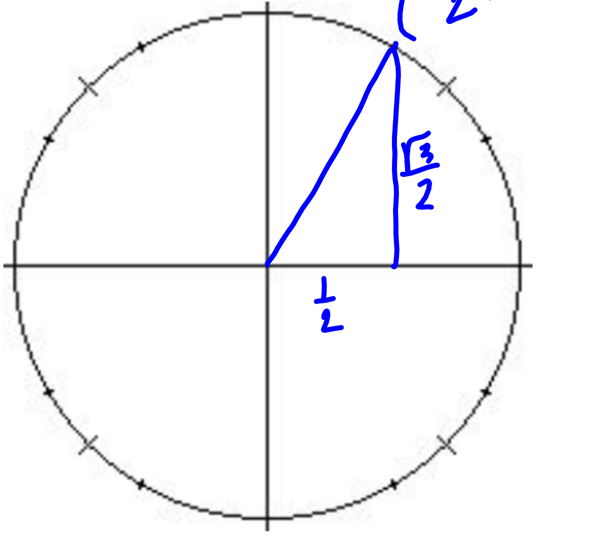


Θ=45°



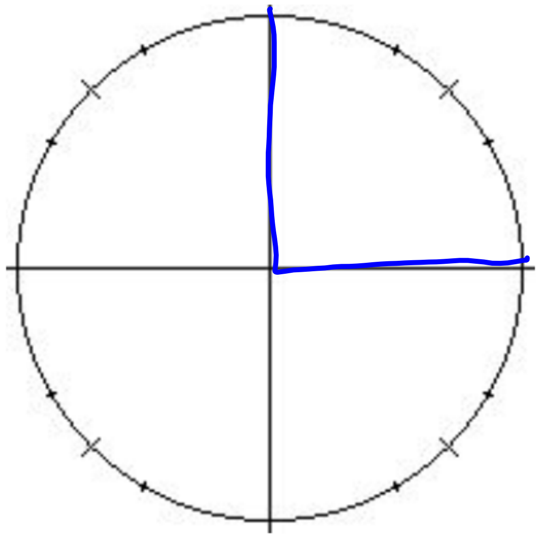
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Θ=60°

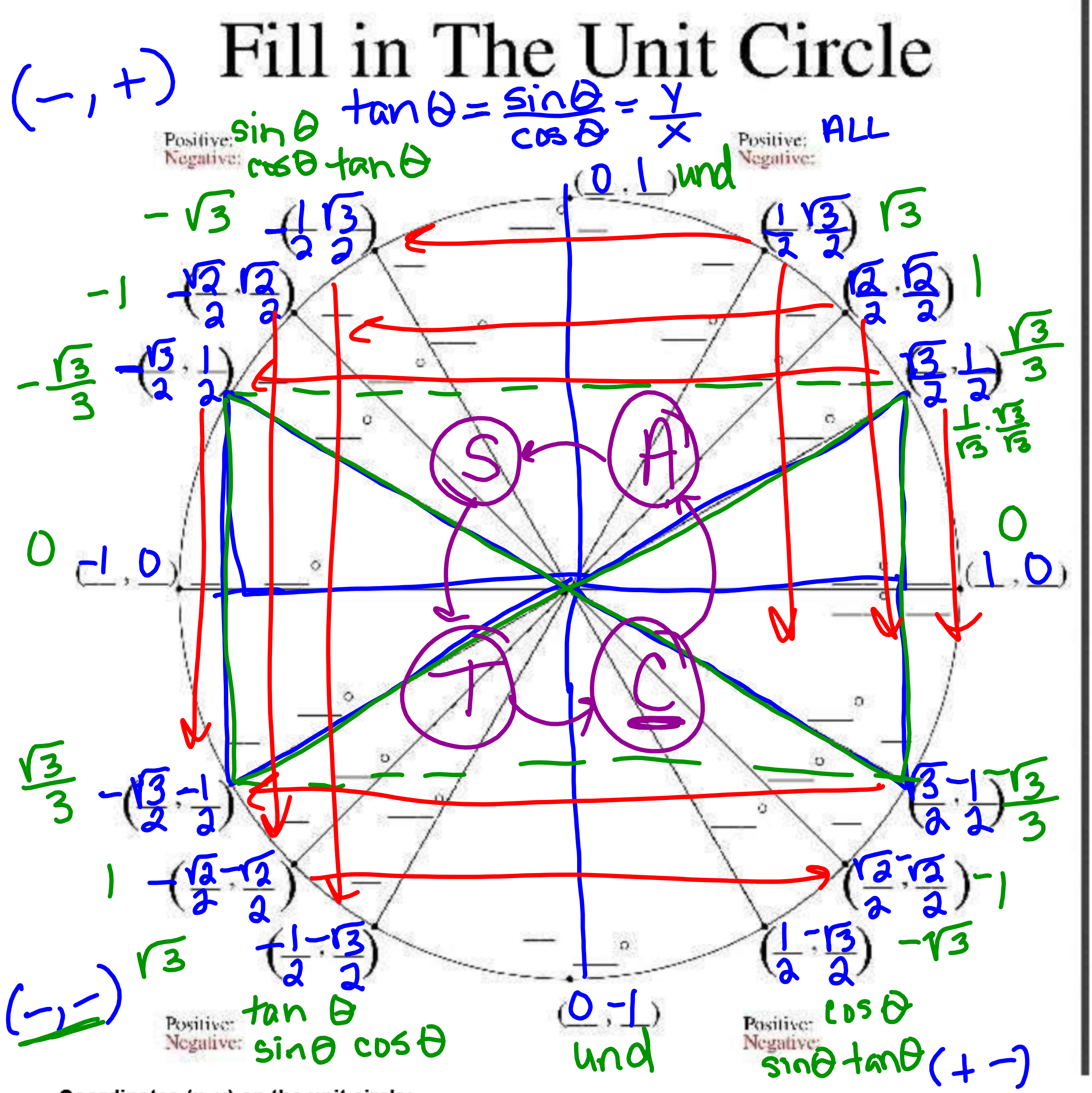


$(0, 1)$

Θ=0° or Θ=90°



Now let's do all four quadrants...



Coordinates (x, y) on the unit circle:

cosΘ = x

sinΘ = y

tanΘ = $\frac{y}{x}$

13.4 The Sine Function (Day 1)

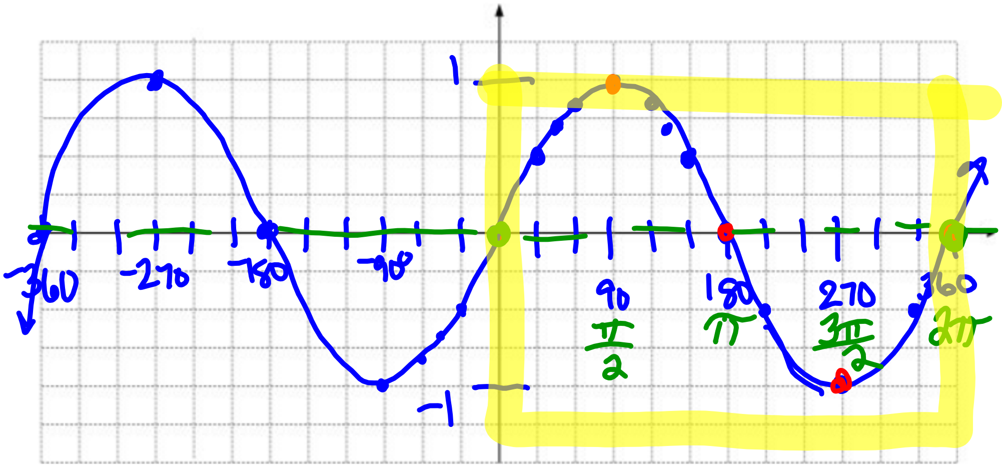
Warm Up: Use the graph. State:

1. the period 360° or 2π
2. the domain ARN
3. the amplitude 1
4. the range $-1 \le y \le 1$ $[-1, 1]$

sine function y=sinΘ: for each measure of Θ, the sine of Θ corresponds with the _y_-coordinates on the unit circle.

y=sinΘ

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Θ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ 0.707	$\frac{\sqrt{3}}{2}$ .866	1	0	-1	0



Example 1: Interpreting the Sine Function in Degrees

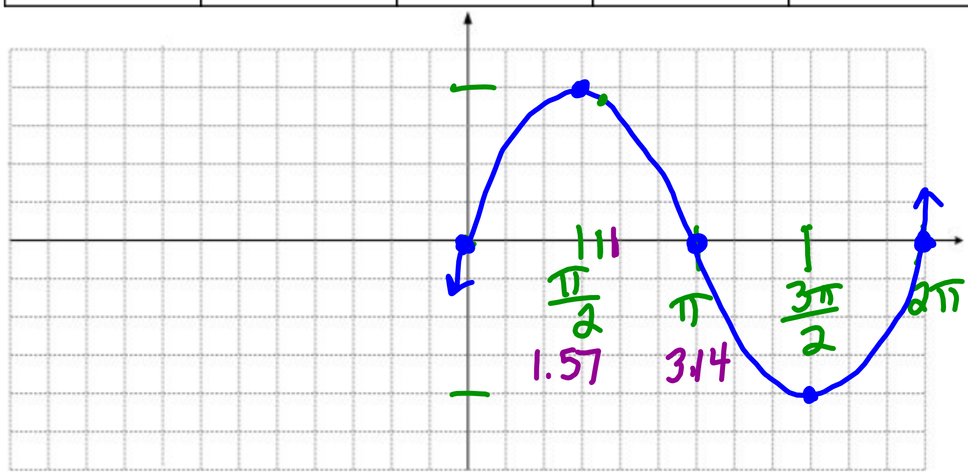
a. What is the value of y=sinΘ for Θ=270°? -1

b. For what values of Θ between 0° and 360° does the graph of y=sinΘ reach
   - the maxiumum value of y=1? 90°
   - the minimum value of y=-1? 270°
   - x-intercept of y=0? aka "zero" 0°, 180°, 360°

Mathematical convention: An angle measure θ can be expressed in degrees or in radians. However, when no unit is mentioned, you should use radians. Let's graph the sine function in radians…

y=sinθ

θ (radians)					
y					



Example 2: Estimating Sine Values in Radians

Use your graph above to estimate the value. Check your estimate with a calculator.

a. sin 2 0.8 0.909

b. sin π 0

For the sine function, find the following:

a. amplitude

1

b. period (in degrees and radians)

$360°$

2π

c. domain and range

D: ARN

R: $[-1, 1]$

from to

$-1 \le y \le 1$

$[0, 5)$

$0 \le y < 5$

Properties of Sine Functions

Suppose **y = a sin bθ**, where a≠0, b>0, and θ in radians.

- The amplitude of the function is $\underline{a}$
- The number of cycles in the interval from 0 to 2π is $\underline{b}$
- The period of the function is $\underline{\frac{2\pi}{b}}$ $\frac{360}{b}$

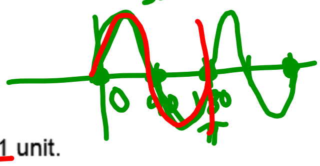
$y = \sin \theta$ 360 period

$\theta \rightarrow 2\pi$

$\sin(2\theta)$ 360

sin 360

sin 180



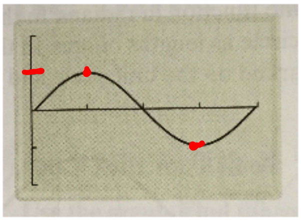
Examples 3&4: Finding the Period and Amplitude of a Sine Function

a. Find the amplitude.

b. How many cycles does the sine function have in the interval from 0 to 2π?

c. Find the period.

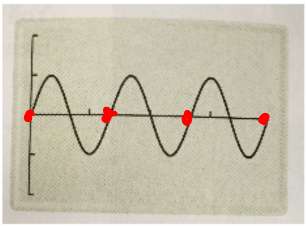
The θ-axis represents values from 0 to 2π. Each interval on the y-axis represents $\underline{1}$ unit.



a : 1

$b\theta$ b : 1

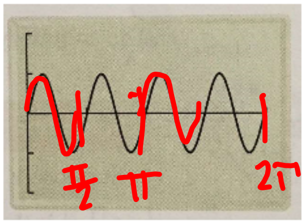
$\frac{2\pi}{b}$ p : 2π



a : 1

b : 3

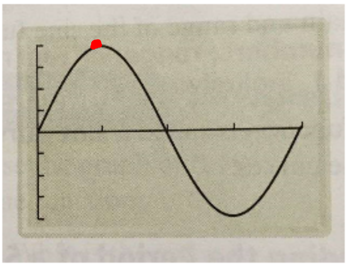
p : $\frac{2\pi}{3}$



a : 1

b : 4

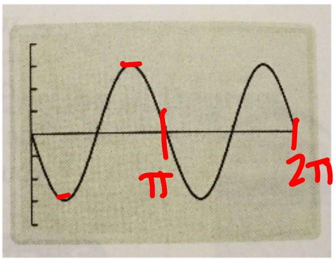
p : $\frac{2\pi}{4} = \frac{\pi}{2}$



a : 4

b = 1

p : 2π



a : 3

b = 2

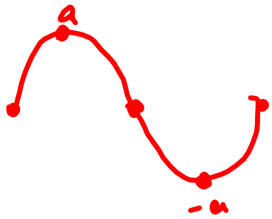
p = $\frac{2\pi}{2} = \pi$

13.4 The Sine Function (Day 2)

II. Graphing Sine Functions

You can use 5 points equally spaced through one cycle to sketch a sine curve. For a>0, this 5-point pattern is zero-max-zero-min-zero.

$y = a \sin b\theta$

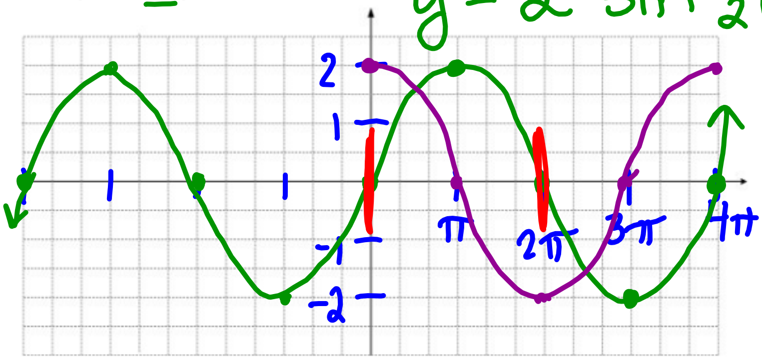


Example 5: Sketching a Graph

Sketch one cycle of each sine curve. Then write an equation for each graph.

a. amplitude 2, period 4π, a>0

$y = 2 \sin \frac{1}{2}\theta$



(1,0)

$\cos 0 = 1$

$\boxed{p = \frac{2\pi}{b}}$

$4\pi = \frac{2\pi}{b}$

$\frac{4\pi b}{4\pi} = \frac{2\pi}{4\pi}$

$b = \frac{1}{2}$

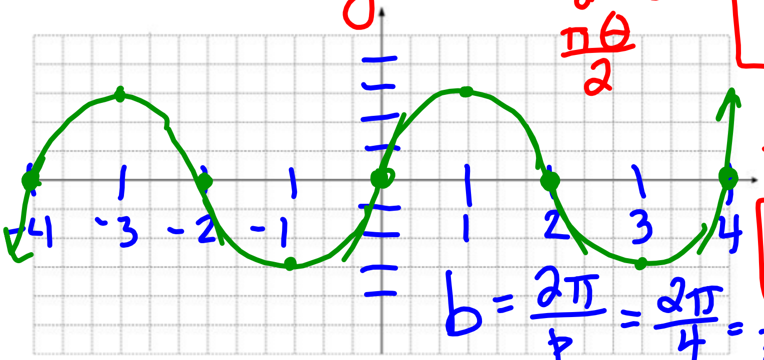
b. amplitude 3, period 4, a>0

$y = 3 \sin \frac{1}{2}\pi\,\theta$

$\frac{\pi\theta}{2}$

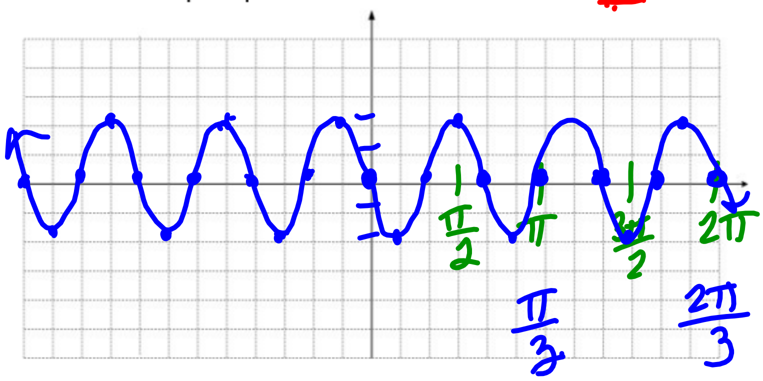
$\frac{bp}{p} = \frac{2\pi}{p}$

$\boxed{b = \frac{2\pi}{p}}$



$b = \frac{2\pi}{p} = \frac{2\pi}{4} = \frac{1}{2}\pi$

c. Predict the 5-point pattern for the sine function when a<0. Then sketch amplitude 2, period 2π/3.



$y = -2 \sin 3\pi$

$b = \frac{\cancel{2\pi}}{\left(\frac{2\pi}{3}\right)} \cdot \frac{3}{\cancel{2\pi}} = 3$

Example 6: Graphing from a Function Rule

Sketch one cycle of the following sine functions.

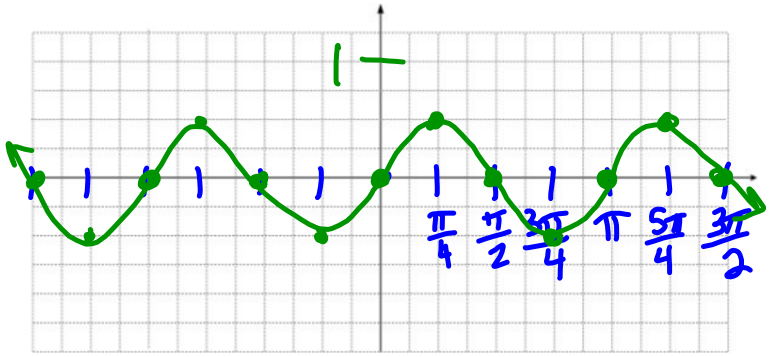
1. $y = \frac{1}{2}sin2\theta$ $b = 2$

amplitude: $\frac{1}{2}$

period: π $p = \frac{2\pi}{b} = \frac{2\pi}{2}$

interval spacing on Θ-axis: $\frac{\pi}{8}$

5-point pattern:



2. $y = 3sin\frac{\pi}{2}\theta$

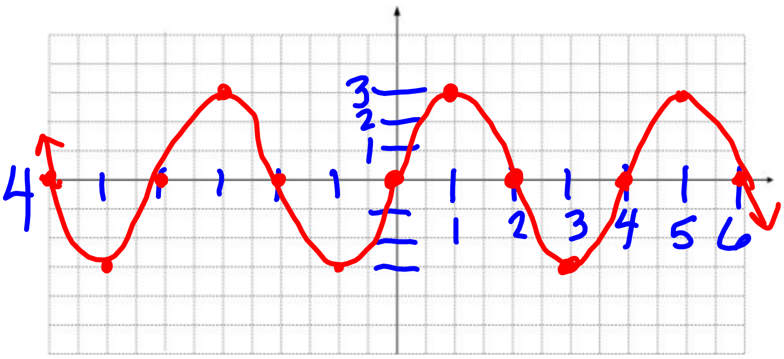
amplitude: 3

period: $\frac{2\pi}{\frac{\pi}{2}} = 2\cancel{\pi} \cdot \frac{2}{\cancel{\pi}} = 4$

$b = \frac{\pi}{2}$

interval spacing on Θ-axis: 0.5

5-point pattern:



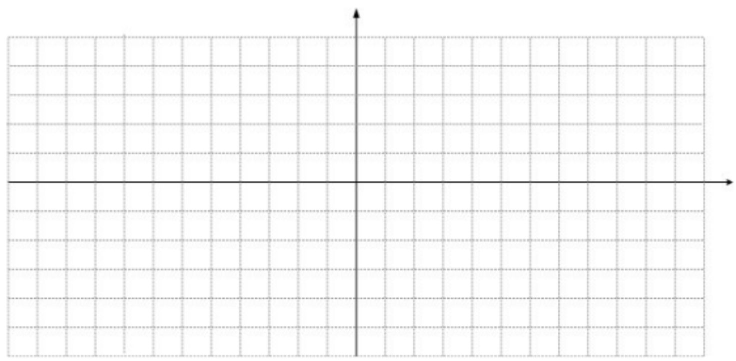
3. $y = -4sin\frac{1}{2}\theta$

amplitude:

period:

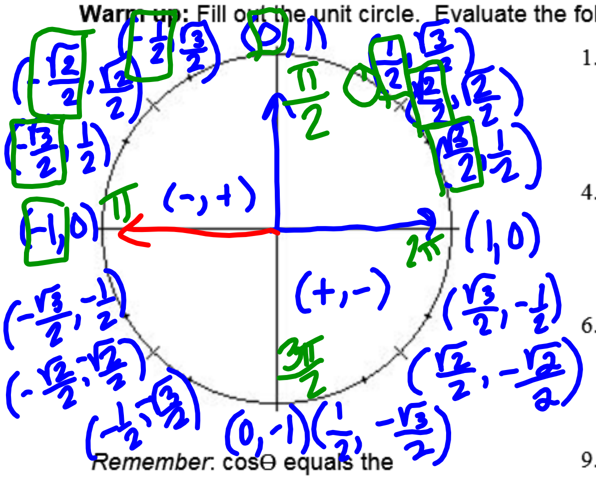
interval spacing on Θ-axis:

5-point pattern:



13.5 The Cosine Function (Day 1)

Warm up: Fill out the unit circle. Evaluate the following angles of cosine.

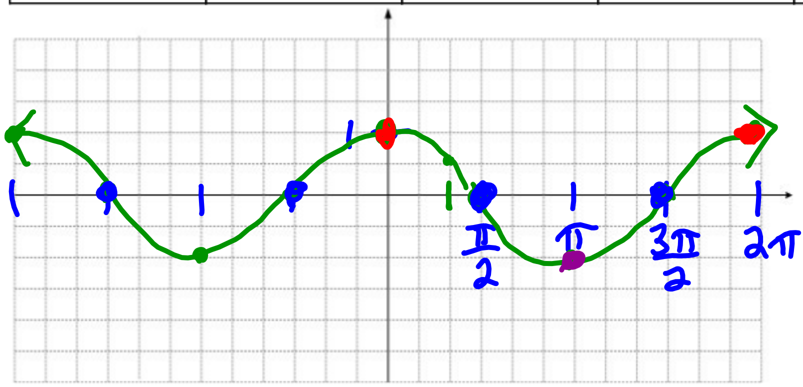


Remember: cosΘ equals the _____-coordinate on the unit circle.

1. $\cos 0^\circ$ 1
2. $\cos 90^\circ$ 0
3. $\cos 180^\circ$ −1
4. $\cos 270^\circ$ 0
5. $\cos 360^\circ$ 1
6. $\cos 0$ 1
7. $\cos \frac{\pi}{2}$ 90° 0
8. $\cos \pi$ −1
9. $\cos \frac{3\pi}{2}$ 0
10. $\cos 2\pi$ 1

I. Graph the cosine function: y=cosΘ

Θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	−1	0	1



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Example 1: Interpreting the Graph of y=cosΘ

a. Use your graph above. Find the following:

domain ARN

period 2π

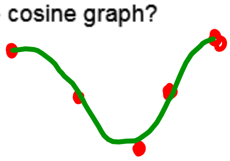
range $[-1, 1]$ $-1 \le y \le 1$

amplitude 1

b. In the interval from 0 to 2π, where do the maximum occur? minimum? zeros?

max: $0, 2\pi$ min: π zeros: $\frac{\pi}{2}, \frac{3\pi}{2}$

c. What is the 5-point pattern of the cosine graph?



Properties of Cosine Functions

Suppose **y = *a* cos *b*θ**, where a≠0, b>0, and θ in radians.

- The amplitude of the function is a
- The number of cycles in the interval from 0 to 2π is b
- The period of the function is $\frac{2\pi}{b}$

Example 2: Sketching the Graph of a Cosine Function

Sketch one cycle of the following cosine functions.

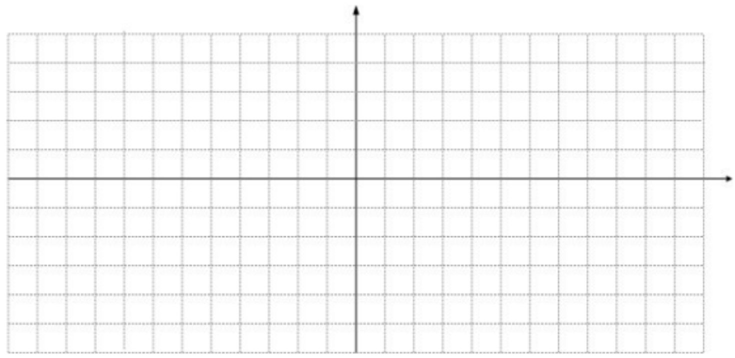
1. $y = cos\frac{\pi}{2}\theta$

amplitude:

period:

interval spacing on θ-axis:

5-point pattern:



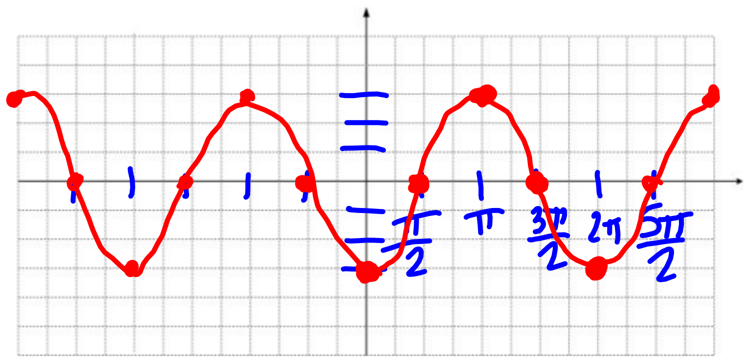
2. $y = -3cos\theta$

amplitude: −3

period: 2π

interval spacing on θ-axis: $\frac{\pi}{4}$

5-point pattern:



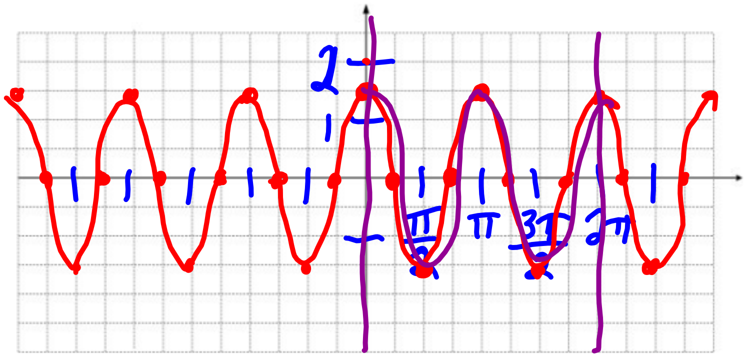
3. $y = 1.5\cos 2\theta$ 2

amplitude: 1.5

period: $\frac{2\pi}{2} = \pi$

interval spacing on Θ-axis: $\frac{\pi}{4}$

5-point pattern:



13.5 The Cosine Function (Day 2)

Warm up: Graph the following cosine functions.

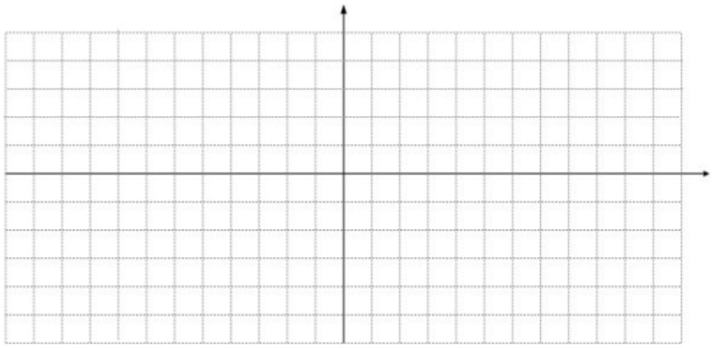
1. $y = 3\cos 2\theta$

amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:



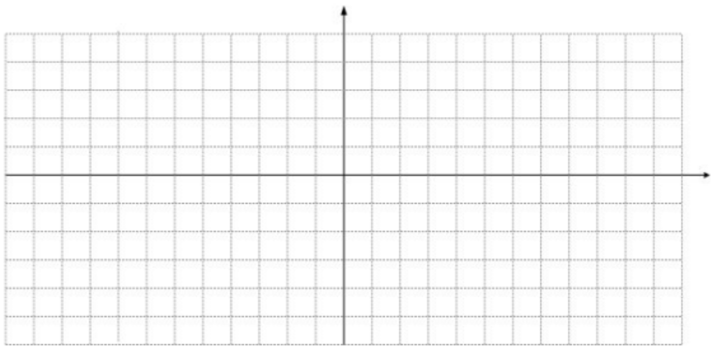
2. $y = -2\cos\theta$

amplitude:

period:

interval spacing on Θ-axis:

5-point pattern:



II. Solving Trigonometric Equations

Example 4: Solving a Cosine Equation

Solve the cosine equation in the interval from 0 to 2π. Round to the nearest hundredth. Calculators needed.

a. $-2\cos\theta = 1.2$

$$\frac{-2\cos\theta}{-2} = \frac{1.2}{-2}$$

$$\cos^{-1}(\cos\theta) = \cos^{-1} -0.6$$

$$\theta = \cos^{-1}(-0.6)$$

-0.6 $\boxed{2nd}$ $\boxed{\cos}$ ($\cos^{-1}$)

$$\theta = 2.2143$$

$$\boxed{2.21}$$

b. $3\cos 2t = -2$

$$\frac{3\cos 2t}{3} = \frac{-2}{3}$$

$$\cos^{-1}\ \cos 2t = \cos^{-1} -\frac{2}{3}$$

2.30052

$$\frac{2t}{2} = \frac{\cos^{-1}\left(-\frac{2}{3}\right)}{2}$$

$$t = 1.15$$

c. $5\cos\frac{\pi}{2}t = 3$

Identify the period, range, and amplitude of each function.

22. $y = 3\cos\theta$

$b = 1$

$a = 3$

$p = 2\pi$

$[-3, 3]$

$-3 \le y \le 3$

24. $y = 2\cos\frac{1}{2}t$

$a = 2$

$b = \boxed{\frac{1}{2}}$

$[-2, 2]$ $-2 \le y \le 2$

$p = 2\pi \cdot \frac{2}{1} = 4\pi$

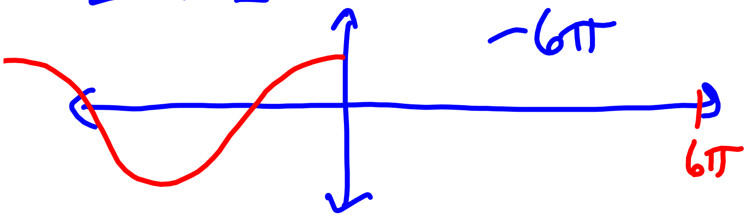
26. $y = 3\cos\left(-\frac{\theta}{3}\right)$

$a = 3$

$[-3, 3]$

$P = \frac{2\pi}{-\frac{1}{3}} = 2\pi \cdot -3$

-6π



28. $y = 16\cos\frac{3\pi}{2}t$

$a = 16$

$[-16, 16]$

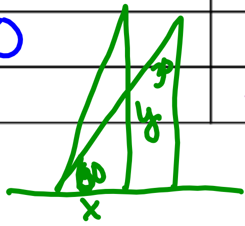
$b = \frac{3\pi}{2}$

$P = \frac{2\pi}{\frac{3\pi}{2}} = 2\cancel{\pi} \cdot \frac{2}{3\cancel{\pi}} = \boxed{\frac{4\pi}{3}}$

13.6 The Tangent Function

Warm up: Use a calculator to find the sine and cosine of each Θ. Then calculate the ratio of sinΘ to cosΘ.

θ	sinθ "y"	cosθ	$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{y}{x}$
1. $\frac{\pi}{3}$ 60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$ 1.732
2. 30° $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ 0.577
3. 90°	1	0	$\frac{1}{0}$ = undefined
4. π	0	-1	$\frac{0}{-1} = 0$
5. $\frac{7\pi}{6} = 1\frac{1}{6}\pi$, $\pi + \frac{1}{6}\pi$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

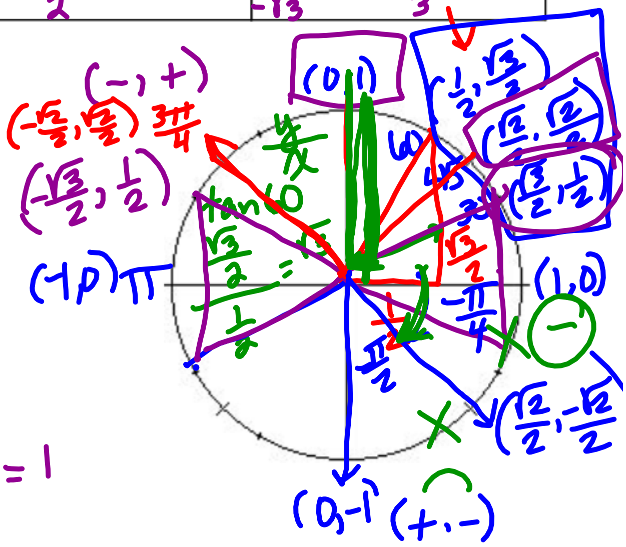


I. The Tangent Function

The cosΘ is derived from the __x__ - coordinate of the point on the unit circle.

The sinΘ is derived from the __y__ - coordinate of the point on the unit circle.

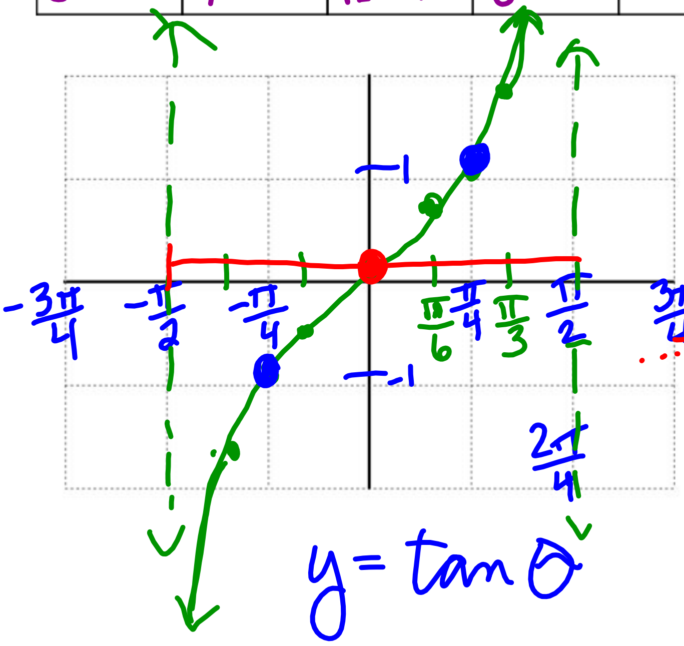
The tanΘ is derived from the ratio of sinΘ to cosΘ. In other words: tanΘ = $\frac{y}{x}$



$$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}\cdot\frac{1}{2}}{\sqrt{2}\cdot\frac{1}{2}} = 1$$

y = tanΘ $\frac{y}{x}$

Θ (radians)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$
y	$\frac{0}{1} = 0$	$\frac{\sqrt{2}}{\sqrt{2}} = 1$	$\frac{1}{0}$ = und.	-1	$\frac{0}{-1} = 0$	-1	$\frac{-1}{0}$ = und.



$y = \tan\theta$

Features of the parent tangent function:

passes through: $(0,0)$ $(\frac{\pi}{4}, 1)$ $(-\frac{\pi}{4}, -1)$

each "branch" is: period: π $(-\frac{\pi}{2}, \frac{\pi}{2})$

x-intercepts (zeros): $\ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots$: $\boxed{n\pi}$ n → integer

vertical asymptotes: $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$ $\frac{\pi}{2} + n\pi$ n → integer

"guide points": $(0,0)$ $(\frac{\pi}{4}, 1)$ $(-\frac{\pi}{4}, 1)$ $-\frac{\pi}{2}$ $\frac{\pi}{2}$

Properties of Tangent Functions

Suppose **y = *a* tan *bΘ***, where b>0, and Θ in radians.

- The period of the function is $\frac{\pi}{b}$
- 1 cycle occurs in the interval from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$
- There are vertical aymptotes at each end of the cycle.
- The pattern is *"asymptote, $\boxed{-a}$ zero $\boxed{a}$ asymptote"*.

Example 2: Graphing a Tangent Function

Sketch 2 cycles of each tangent function.

1. $y = \tan \pi\theta$ $b = \pi$

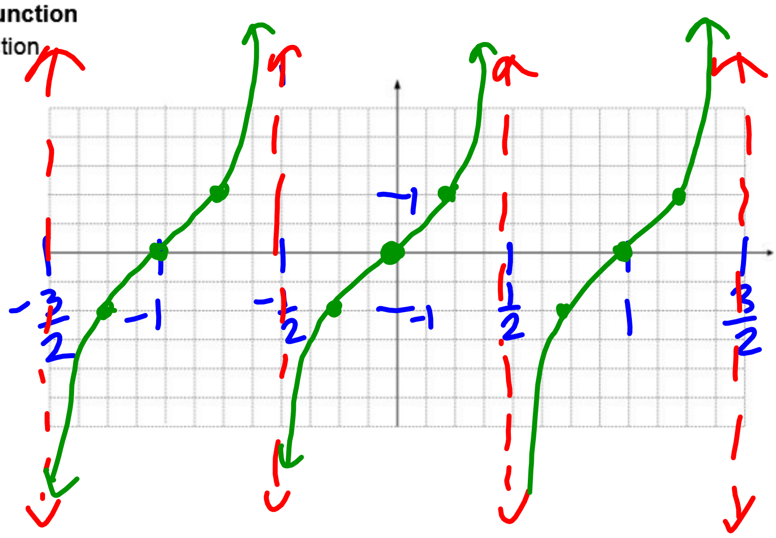
period: $\frac{\pi}{\pi} = \boxed{1}$

1 cycle: from $-\frac{1}{2}$ to $\frac{1}{2}$

VA: $\frac{1}{2} + 1n$

2 guide points: $(-\frac{1}{4}, -1), (\frac{1}{4}, 1)$

pattern:



$a = 1, \ b = 3$

2. $y = \tan 3\theta$

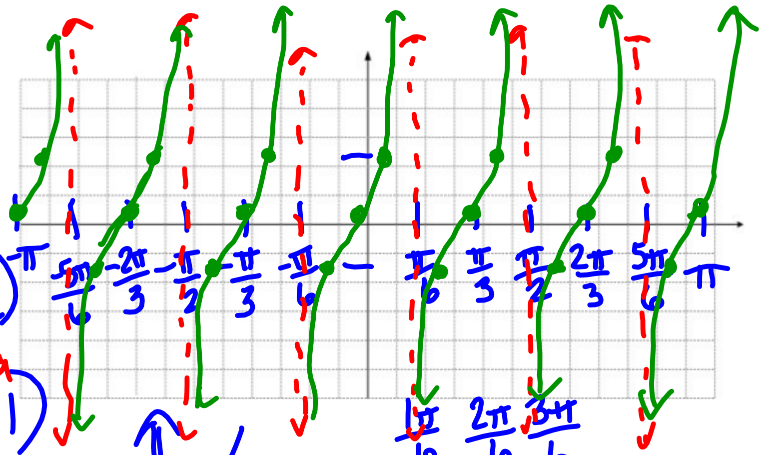
$\frac{\pi}{b}$ period: $\frac{\pi}{3}$ $(\cdot \frac{1}{2})$

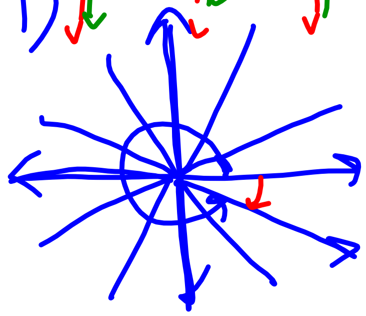
$\frac{\pi}{6} \cdot \frac{1}{2}$ 1 cycle: from $-\frac{\pi}{6}$ to $\frac{\pi}{6}$

VA: $x = -\frac{\pi}{6}, \frac{\pi}{6}$ $(\cdot\frac{1}{2})$

$\cdot \frac{1}{2}$ 2 guide points: $(-\frac{\pi}{12}, -1)$ $(\frac{\pi}{12}, 1)$

pattern: $-a, 0, a$





$\frac{11\pi}{6}$ $\frac{12\pi}{6} = 2\pi$

$\frac{11\pi}{6}$ $-\frac{\pi}{6}$

3. $y = \tan\frac{\pi}{2}\theta$

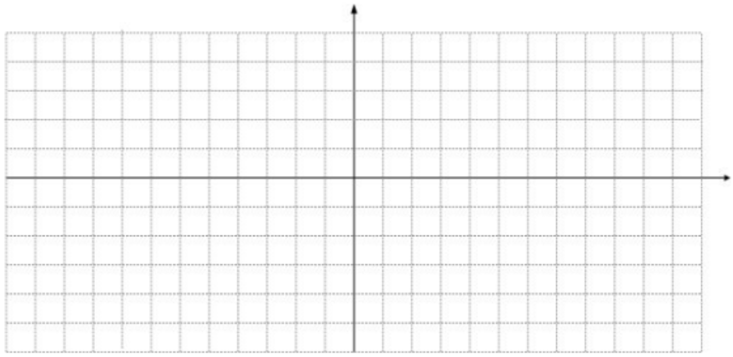
period:

1 cycle: from ______ to ______

VA:

2 guide points:

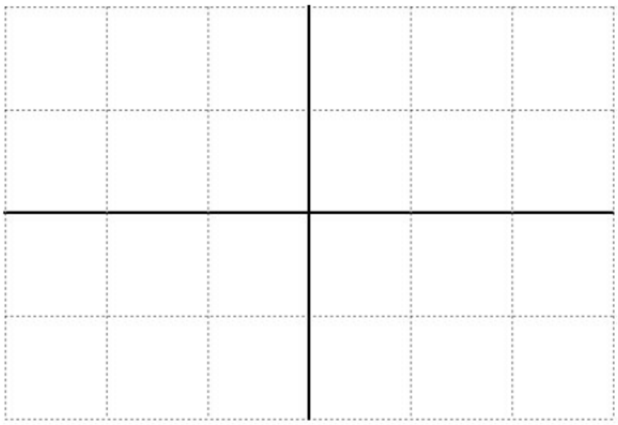
pattern:



13.7 Translating Sine and Cosine Functions

Warm Up:

1. Graph y=tanΘ ... again! (Try not to peek at prior notes.)



Features of the tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

"guide points":

2. Compare each pair of equations. State the translations (horizontal, vertical) involved.

left/right up/down

a. $y = 2x,\ y = 2x + \underline{5}$

vertical translation 5 up

b. $y = |x|,\ y = |x \underline{+ 3}|$

left 3

c. $y = x^2,\ y = x^2 \underline{- 4}$

down 4

d. $y = |x-2| + 1$

right 2 up 1

e. $y = x^2,\ y = (x+3)^2 - 6$ v: (-3,-6)

left 3 down 6

d. $y = f(x),\ y = f(x-h) + k$

horizontal h vertical k

I. Graphing Translations of Trigonometric Functions

Phase shift - the horizontal translation of a function. If *f(x)* is the "parent", then *f(x-h)* translates horizontally *h* units. For example: *f(x-1)* translates right 1 unit, *f(x+3)* translates left 1 unit

Vertical shift - the vertical translation of a function. If *f(x)* is the "parent", then *f(x)+k* translates vertically k units. For example: *f(x) - 1* translates down 1 unit, *f(x) + 3* translates up 3 units

Example 1: Identifying Phase Shifts and Vertical Shifts

What is the value of *h* and *k* in each translation? Describe the shift i.e. "3 units to the left".

a. f(x-2)

right 2 units

b. y= cos(x+4) (0,1)

left 4 (-4,1)

c. f(t-5)

right 5

d. y= sin(x+3) (0,0)

left 3 (-3,0)

e. f(x) - 2

down 2

b. y= cosx + 4

up 4

(0,1)→(0,5)

c. f(t) - 5

down 5

d. y= sinx + 3

up 3 (0,0) (0,3)

Example 2: Graphing Translations

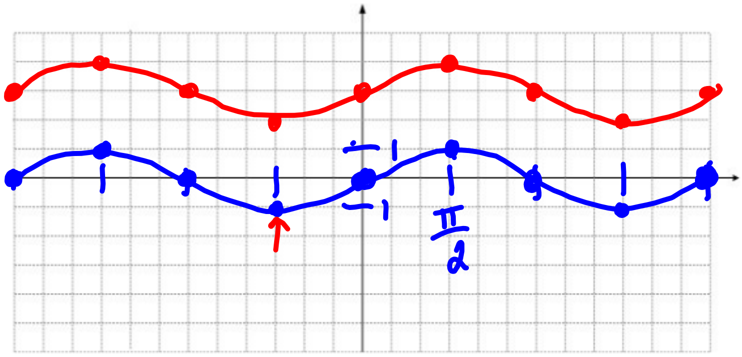
Make a table and then graph the following functions on the same set of axes:

y= sinx

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	−1	0
	+3	+3	+3	+3	+3

y= sinx + 3

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	3	4	3	2	3

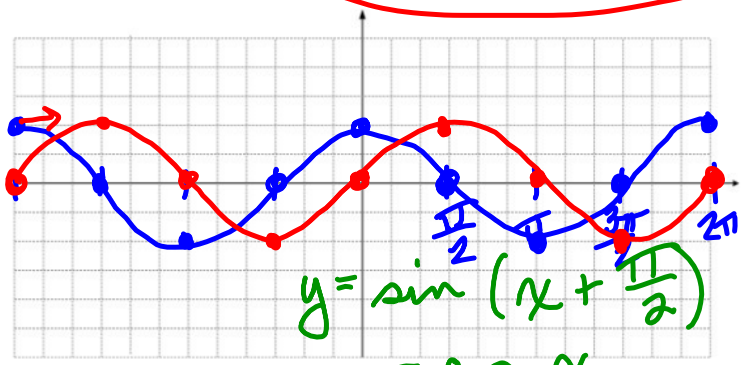


$y = \cos x$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	−1	0	1

$y = \cos(x - \boxed{\frac{\pi}{2}})$ right $\frac{\pi}{2}$

x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
y	1	0	−1	0	1



$y = \sin\left(x + \frac{\pi}{2}\right)$
$= \cos x$

$y = \cos\left(x - \frac{\pi}{2}\right)$

$y = \sin x$

Example 3: Graphing a Combined Translation

1. $y = sin(x + \pi) - 2$

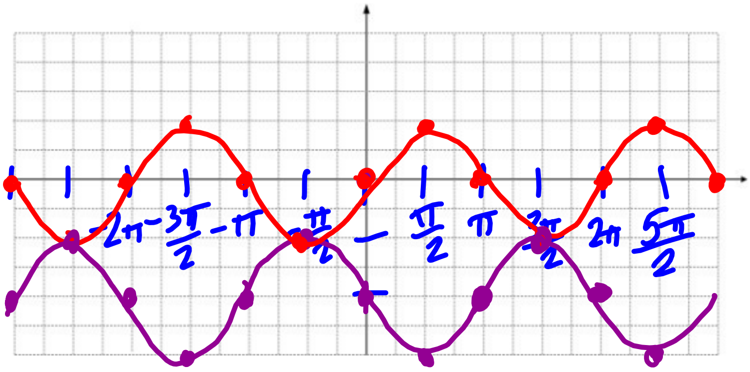
amplitude: 1

period: 2π

phase shift: left π

interval spacing on x-axis:

vertical shift: down 2



5 point pattern:

2. $y = 2cos(x - \frac{\pi}{2}) + 3$

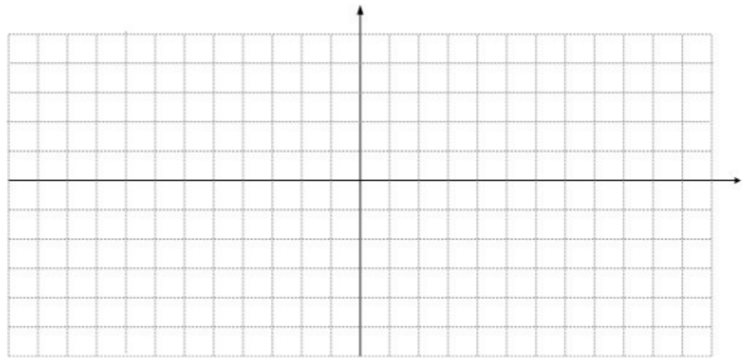
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

<u>Summary: Families of Sine and Cosine Functions</u>

Parent **Transformed Function**

y=sinx ____________________

y=cosx ____________________

amplitude = h =

period = k =

13.7 Translating Sine and Cosine Functions (Day 2)
Warm Up:

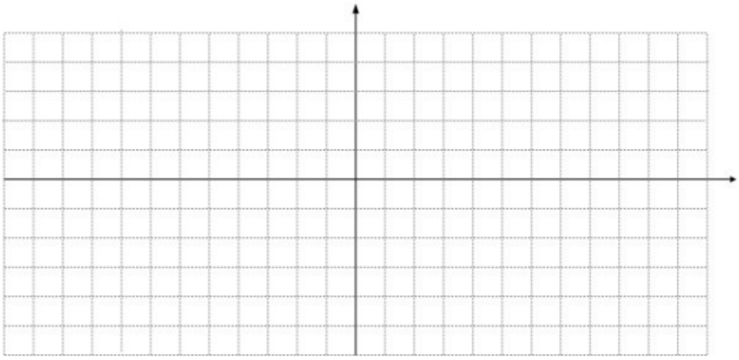
$y = -3sin(x + \frac{\pi}{2}) + 2$

amplitude:

period:

phase shift:

interval spacing on x-axis:



vertical shift:

5 point pattern:

When the phase shift is a pesky number...

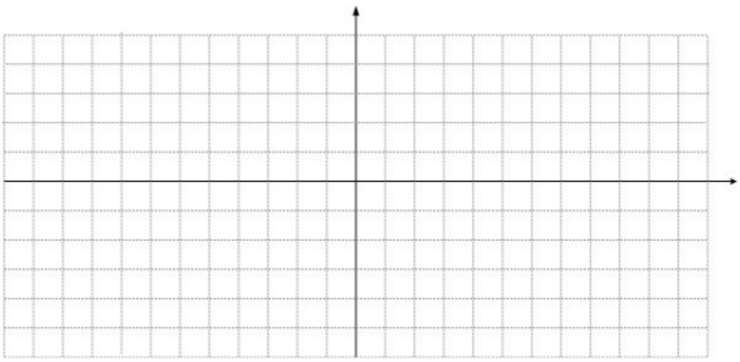
$y = sin(x - \frac{\pi}{3})$

amplitude:

period:

phase shift:

interval spacing on x-axis:



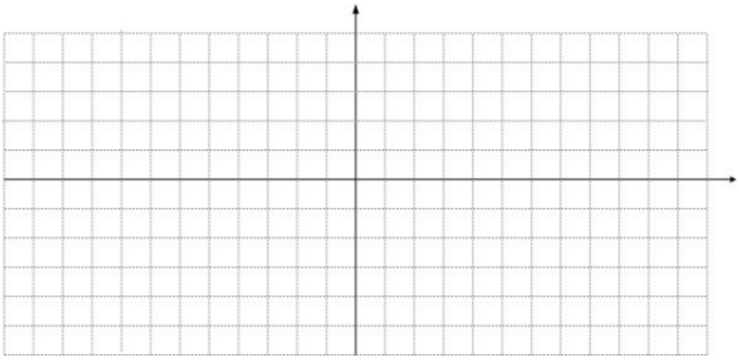
vertical shift:

5 point pattern:

Graphing a translation of y = sin 2x...

$y = sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

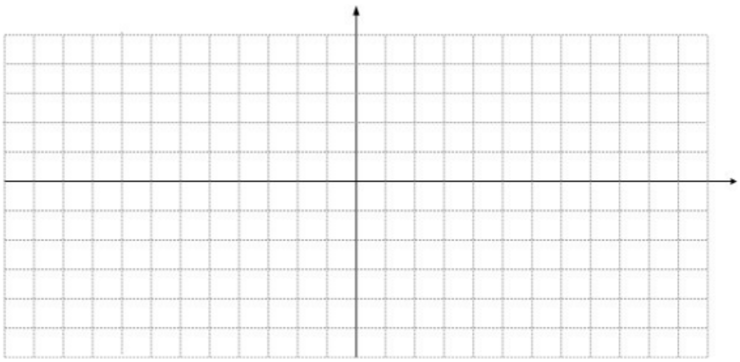
interval spacing on x-axis:

vertical shift:

5 point pattern:

$y = -3sin2(x - \frac{\pi}{3}) - \frac{3}{2}$

amplitude:



period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:

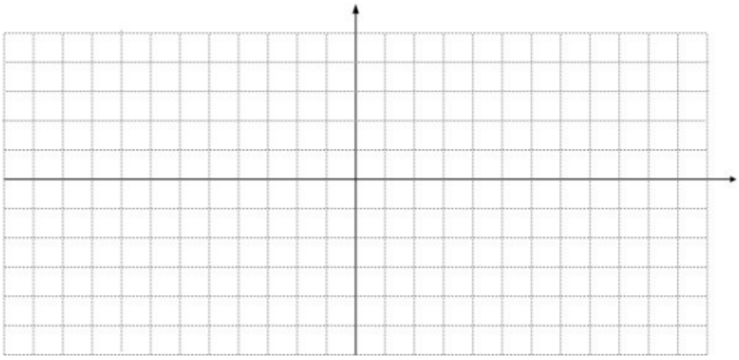
$y = 2\cos\frac{\pi}{2}(x+1) - 3$

amplitude:

period:

phase shift:

interval spacing on x-axis:



vertical shift:

5 point pattern:

Example 5: Writing a Translation

Write an equation for each translation.

a. $y = \sin\pi,\ \ \pi\ units\ down$

b. $y = -\cos x,\ \ 2\ units\ left$

c. $y = \cos x,\ \ \frac{\pi}{2} units\ up$

$y = \cos(x) + \frac{\pi}{2}$

up

d. $y = 2\sin x,\ \ \frac{\pi}{4} units\ right$

$y = 2\sin\left(x - \frac{\pi}{4}\right)$

right

$\frac{O}{H} \leftrightarrow \frac{H}{O}$

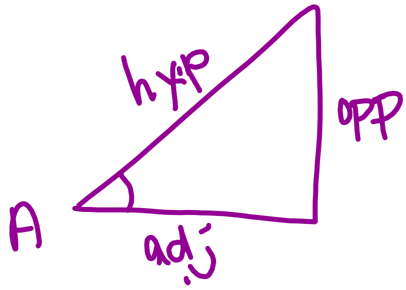
SOH CAH TOA
CHO SHA CAO

13.8 Reciprocal Trigonometric Functions
Warm up:
Find the reciprocal of each fraction:

1. $\frac{9}{13}$ $\quad \frac{13}{9}$
2. $-\frac{5}{8}$
3. $\frac{1}{2\pi}$
4. $\frac{14}{-t}$
5. θ

Name the 3 trigonometric functions you have studied so far:

1. ___________
2. ___________
3. ___________



These 3 trigonometric functions have reciprocals.

Definition: Cosecant, Secant, and Cotangent

Example 1: Using Reciprocals
a. Use your calculator (degree mode). Round your answer to the nearest hundredth.

csc 60° cot 55° sec 15°

b. Suppose $cos\theta = \frac{5}{13}$. Find $sec\theta$.

c. Suppose $sin\theta = \frac{-12}{13}$. Find $csc\theta$.

Example 2: Find The Exact Value

csc 30° csc 45° csc 60° csc 90°

sec 30° sec 45° sec 60° sec 90°

cot 30° cot 45° cot 60° cot 90°

Example 3: Using Radians

a. Use your calculator (radian mode). Round your answer to the nearest hundredth.

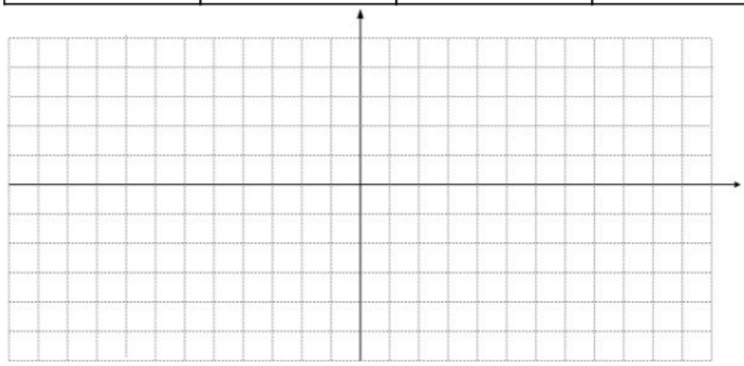
$sec(-1)$ $csc(-1.5)$ $sec2$

b. Find the exact value.

$cot\frac{\pi}{3}$ $cot\pi$ $sec0$

Example 4: Graph The Reciprocal Trigonometric Functions

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= sin x					
y= csc x					



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y= cos x					
y= sec x