

# Sec. 8.5 Adding and Subtracting Rational Expressions

Problem 1:

$$\begin{array}{cc} 4 & 6 \\ \textcircled{2} \textcircled{2} & \textcircled{2} \textcircled{3} \end{array}$$

What is the LCM of  $LCM: 2 \cdot 2 \cdot 3 = 12$

$$x^2 + 4x - 12 \begin{array}{l} \frac{1 \cdot 12}{2 \cdot 6} \\ \frac{3 \cdot 4}{} \end{array} \text{ and } x^2 - 6x + 8 \begin{array}{l} \frac{1 \cdot 8}{2 \cdot 4} \end{array}$$

$$\begin{array}{l} (x+6)(x-2) \\ (x-4) \end{array} \quad \begin{array}{l} (x-2)(x-4) \\ (x+6) \end{array}$$

$$LCM: (x+6)(x-2)(x-4)$$

- ① Factor each
- ② Write factors of first
- ③ Write any additional factors of second

b.  $2x + 4$  and  $x^2 - x - 6$   $\begin{array}{l} \frac{1 \cdot 6}{2 \cdot 3} \end{array}$

$$2(x+2)(x-3) \quad 2(x+2)(x-3)$$

$$LCM: 2(x+2)(x-3)$$

c.  $x^2 + 3x - 4$   $\begin{array}{l} \frac{1 \cdot 4}{2 \cdot 2} \end{array}$   $x^2 + 2x - 8$   $\begin{array}{l} \frac{1 \cdot 8}{2 \cdot 4} \end{array}$   $x^2 - 4x + 4$   $\begin{array}{l} \frac{1 \cdot 4}{2 \cdot 2} \end{array}$

$$\begin{array}{l} (x-1)(x+4) \\ (x-2)(x-2) \end{array} \quad \begin{array}{l} (x-2)(x+4) \\ (x-2)(x+1) \end{array} \quad \begin{array}{l} (x-2)(x-2) \\ (x-1)(x+4) \end{array}$$

$$LCM: (x-1)(x+4)(x-2)^2$$

Problem 2:

$$\frac{2x}{2x} \cdot \frac{1}{2x} + \frac{1}{4x^2}$$

$$a. \quad \frac{4(x+3)}{x^2+3x} + \frac{(x-2)x}{x^2+6x+9} = 4x+12+x^2-2x$$

$$x(x+3)(x+3) \quad (x+3)(x+3)x$$

$$\frac{x^2+2x+12}{x(x+3)^2}, \quad x \neq 0, -3$$

$$b. \quad \frac{5x(x+2)}{(x^2-x-6) \cdot \frac{1 \cdot 6}{2 \cdot 3}} + \frac{4(x-3)}{(x^2+4x+4) \cdot \frac{1 \cdot 4}{2 \cdot 2}}$$

$$(x+2)(x-3)(x+2) \quad (x+2)(x+2)(x-3)$$

$$\frac{5x^2+10x+4x-12}{(x+2)^2(x-3)}$$

$$\begin{array}{r} a.c \\ -6 \cdot 0 \\ -1 \cdot 6 \\ -2 \cdot 3 \\ -3 \cdot 2 \\ -4 \cdot 1 \\ -5 \cdot 1 \\ -6 \cdot 0 \end{array}$$

$$\frac{5x^2+14x-12}{(x+2)^2(x-3)}, \quad x \neq -2, 3$$

Problem 3:

$$a. \quad \frac{4(x+1)}{x^2+2x-8} \oplus \frac{-x(x+4)}{4x-8}$$

$$4(x-2)(x+4) \qquad 4(x-2)(x+4)$$

$$\frac{4x+4-x^2-4x}{4(x-2)(x+4)} = \frac{-x^2+4}{4(x-2)(x+4)}, x \neq 2, -4$$

$$-x^2+4 = -(x^2-4) = \frac{-(x+2)\cancel{(x-2)}}{4\cancel{(x-2)}(x+4)}$$

$$= \frac{-(x+2)}{4(x+4)}, x \neq 2, -4$$

Problem 4:

Simplify 
$$\frac{3x - \frac{1}{y}}{\frac{y^2}{x} + x}$$

Method 1: 
$$\left( \frac{3xy - \frac{1}{y}}{y} \right) \div \left( \frac{y^2}{x} + \frac{x \cdot x}{x} \right)$$

$$\frac{3xy - 1}{y} \div \frac{y^2 + x^2}{x}$$

$$\left( \frac{3xy - 1}{y} \right) \cdot \frac{x}{(y^2 + x^2)} = \frac{x(3xy - 1)}{y(y^2 + x^2)}$$

$$\frac{3x^2y - x}{y^3 + x^2y}$$

Method 2: Multiply by LCD of all denominators

$$\frac{xy(3x - \frac{1}{y})}{xy(\frac{y^2}{x} + x)}$$

LCD:  $xy$   
(LCM):

$$\frac{xy \cdot 3x - \frac{1}{y} \cdot xy}{xy \cdot \frac{y^2}{x} + x \cdot xy} = \frac{3x^2y - x}{y^3 + x^2y}$$

$$b. \frac{x+3}{x^2-2x+1} + \frac{x}{x^2-3x+2}$$

$$\frac{x}{x^2-4x+4} - \frac{2}{x^2-4}$$

$$\frac{x+3}{(x-1)(x-1)} + \frac{x}{(x-1)(x-2)}$$

$$\frac{x}{(x-2)(x-2)} - \frac{2}{(x+2)(x-2)}$$

$$\frac{x^2-2x+3x-6}{(x+3)(x-2)} + \frac{x^2-x}{x(x-1)}$$

$$\frac{x(x+2)}{(x-2)(x-2)(x+2)} - \frac{2(x-2)}{(x+2)(x-2)(x+2)}$$

$$\frac{2x^2-6}{(x-1)(x-1)(x-2)} \div \frac{x^2+4}{(x+2)(x-2)(x-2)}$$

$$\frac{(2x^2-6)}{(x-1)(x-1)(x-2)} \cdot \frac{(x+2)(x-2)(x-2)}{(x^2+4)}$$

$$\frac{(2x^2-6)(x+2)(x-2)}{(x-1)(x-1)(x^2+4)}$$

$$x \neq 1, -2, 2$$

b. 
$$\frac{x}{\frac{1}{x} + \frac{1}{y}}$$

Method 1  $\frac{1 \text{ fraction}}{1 \text{ fraction}}$   
add/subtract,  
then divide

$$x \div \left( \frac{1}{x} + \frac{1}{y} \right)$$

$$x \div \left( \frac{y+x}{xy} \right)$$

$$x \cdot \frac{xy}{y+x} = \frac{x^2 y}{y+x}, \quad \begin{array}{l} x \neq -y \\ x \neq 0, \\ y \neq 0 \end{array}$$

$$\begin{array}{r} y+x=0 \\ -y \quad -y \\ \hline x = -y \end{array}$$

Method 2  $\times \text{LCD}$

$$\frac{x \cdot xy}{xy \cdot \frac{1}{x} + \frac{1}{y} \cdot xy} \quad \text{LCD: } xy$$

$$\frac{x^2 y}{y+x}, \quad \begin{array}{l} x \neq -y, \\ x \neq 0, \\ y \neq 0 \end{array}$$

C.

$$\frac{\frac{x^2 + x - 2x - 2 + 2x}{(x-2)(x+1)} + \frac{2x}{(x+1)x}}{x(x+1)} = \frac{\frac{(x+2)(x-1)}{x^2 + x - 2} \cdot \frac{2x}{x(x+1)}}{x(x+1)}$$

$$\frac{\frac{3(x+1)}{(x-1)(x+1)} + \frac{-1(x-1)}{(x+1)(x-1)}}{(x+1)(x-1)} = \frac{2x+4}{(x+1)(x-1)}$$

$$3x+3-x+1$$

$$\frac{\frac{(x+2)(x-1)}{x(x+1)} \cdot \frac{(x+1)(x-1)}{2(x+2)}}{2x} = \frac{(x-1)^2}{2x},$$

$x \neq 0, 1, -1$

