Problem 4:

Simplify $\dfrac{3x - \frac{1}{y}}{\frac{y^2}{x} + x}$

Method 1: $\left(\dfrac{3xy}{y} - \dfrac{1}{y}\right) \div \left(\dfrac{y^2}{x} + \dfrac{x \cdot x}{x}\right)$

$\dfrac{3xy-1}{y} \div \dfrac{y^2+x^2}{x}$

$\dfrac{(3xy-1)}{y} \cdot \dfrac{x}{(y^2+x^2)}$ $\qquad \dfrac{x(3xy-1)}{y(y^2+x^2)}$

$\dfrac{3x^2y - x}{y^3 + x^2y}$

Method 2: Multiply by LCD of all denominators

$\dfrac{xy\left(3x - \frac{1}{y}\right)}{xy\left(\frac{y^2}{x} + x\right)}$ $\qquad$ LCD: xy (LCM):

$\dfrac{xy\,3x - \frac{1}{y} \cdot xy}{xy \cdot \frac{y^2}{x} + x \cdot xy} = \dfrac{3x^2y - x}{y^3 + x^2y}$

Feb 12-8:40 AM

b. $\dfrac{\dfrac{x+3}{x^2-2x+1}+\dfrac{x}{x^2-3x+2}}{\dfrac{x}{x^2-4x+4}-\dfrac{2}{x^2-4}}$ (1·1, 1·2, 1·4, 2·2)

$\dfrac{\dfrac{x+3}{(x-1)(x-1)}+\dfrac{x}{(x-1)(x-2)}}{\dfrac{x}{(x-2)(x-2)}-\dfrac{2}{(x+2)(x-2)}}$

$\dfrac{x^2-2x+3x-6}{(x+3)(x-2)}+\dfrac{x^2-x}{x(x-1)}$

$\dfrac{(x-1)(x-1)(x-2)}{(x-1)(x-2)(x-1)}$

$\dfrac{x(x+2)}{(x-2)(x-2)(x+2)}-\dfrac{2(x-2)}{(x+2)(x-2)(x-2)}$

$x^2+2x-2x+4$

$\dfrac{2x^2-6}{(x-1)(x-1)(x-2)} \div \dfrac{x^2+4}{(x+2)(x-2)(x-2)}$

$\dfrac{(2x^2-6)}{(x-1)(x-1)(x-2)} \cdot \dfrac{(x+2)(x-2)(x-2)}{(x^2+4)}$

x^2-4

$\dfrac{(2x^2-6)(x+2)(x-2)}{(x-1)(x-1)(x^2+4)}$

$x \neq 1, -2, 2$

Feb 12-8:50 AM

b. $\dfrac{x}{\left(\frac{1}{x}+\frac{1}{y}\right)}$

Method 1 $\frac{1 \text{ fraction}}{1 \text{ fraction}}$ add/subtract, then divide

$x \div \left(\frac{1}{x}\cdot\frac{y}{y}+\frac{1}{y}\cdot\frac{x}{x}\right)$

$x \div \left(\frac{y+x}{xy}\right)$

$\frac{x}{1}\cdot\frac{xy}{(y+x)}$

Method 2 $\times$ LCD

$\left(\frac{x}{\frac{1}{x}+\frac{1}{y}}\right)\frac{xy}{xy}$

LCD: xy

$\frac{x\cdot xy}{\frac{1}{x}\cdot xy+\frac{1}{y}\cdot xy}$

$\frac{x^2y}{y+x}$, $x\neq -y$, $x\neq 0$, $y\neq 0$

$\frac{x^2y}{y+x}$, $x\neq y$, $x\neq 0$, $y\neq 0$

$y+x=0$

$-y \quad -y$

$x=-y$

C. $\dfrac{\frac{x-2}{x}+\frac{2}{x+1}}{\frac{3}{x-1}-\frac{1}{x+1}}$ LCD: $x(x+1)(x-1)$

Method 1:

$\dfrac{(x-2)(x+1)}{x(x+1)}+\dfrac{2x}{(x+1)(x)}$

$\dfrac{3(x+1)-1(x-1)}{(x-1)(x+1)}$

$\dfrac{x^2+1x-2x-2+2x}{x(x+1)}$

$\dfrac{3x+3-1x+1}{(x-1)(x+1)}$

$\dfrac{x^2+x-2}{x(x+1)} \div \dfrac{2x+4}{(x-1)(x+1)}$

$\dfrac{x^2+x-2}{x(x+1)} \cdot \dfrac{(x-1)(x+1)}{2x+4}$

$\dfrac{(x-1)(x+2)}{x(x+1)} \cdot \dfrac{(x-1)(x+1)}{2(x+2)}$

$\dfrac{(x-1)^2}{2x}, x \neq 0, -1, -2, 1$

Method 2:

$\dfrac{\frac{(x-2)}{x}\cdot x(x+1)(x-1)+\frac{2x(x+1)(x-1)}{(x+1)}}{\frac{3}{(x-1)}(x)(x+1)(x-1)-\frac{1}{(x+1)}(x)(x+1)(x-1)}$

$\dfrac{(x-2)(x^2-1)+2x(x-1)}{3x(x+1)-1x(x-1)}$

$\dfrac{x^3-1x-2x^2+2+2x^2-2x}{3x^2+3x-x^2+x}$

$(x+2)(x^2-2x+1)$

$\dfrac{x^3-3x+2}{2x^2+4x} = \dfrac{(x+2)(x-1)^2}{2x(x+2)}$

$-2 \mid 1 \quad 0 \quad -3 \quad 2$

$\quad\quad -2 \quad 4 \quad -2$

$1 \quad -2 \quad 1 \quad 0$

$\dfrac{(x-1)^2}{2x}$

c. $\dfrac{\left(\dfrac{x-2}{x}+\dfrac{2}{x+1}\right)}{\left(\dfrac{3}{x-1}-\dfrac{1}{x+1}\right)}$ ←

$\left(\dfrac{(x-2)(x+1)}{x(x+1)}+\dfrac{2x}{(x+1)x}\right)\div\left(\dfrac{3(x+1)}{(x-1)(x+1)}\oplus\dfrac{-1(x-1)}{(x+1)(x-1)}\right)$

$\left(\dfrac{x^2+1x-2x-2+2x}{(x-2)(x+1)+2x}{x(x+1)}\right)\div\left(\dfrac{3x+3-1x+1}{3(x+1)-1(x-1)}{(x-1)(x+1)}\right)$

$\left(\dfrac{x^2+1x-2}{x(x+1)}\right)\div\left(\dfrac{2x+4}{(x-1)(x+1)}\right)$

$\left(\dfrac{(x-1)(x+2)}{x(x+1)}\right)\div\left(\dfrac{2(x+2)}{(x-1)(x+1)}\right)$

$\dfrac{(x-1)(x+2)}{x(x+1)}\cdot\dfrac{(x-1)(x+1)}{2(x+2)}=\dfrac{(x-1)^2}{2x}, x\neq 0,-2,-1,1$

d. $\dfrac{\frac{2}{x+4}+2}{1+\frac{3}{x+4}}$

Method 2: Find LCD, Multiply EVERY term by LCD Simplify

LCD: $(x+4)$

Method 1:

$$\left(\frac{2}{(x+4)}+\frac{2(x+4)}{(x+4)}\right)\div\left(\frac{1(x+4)}{(x+4)}+\frac{3}{(x+4)}\right)$$

$$\frac{2+2x+8}{2+2(x+4)}\div\frac{x+4+3}{x+4}$$

$$\frac{2x+10}{x+4}\div\frac{x+7}{x+4}$$

$$\frac{2(x+5)}{(x+4)}\cdot\frac{(x+4)}{(x+7)}=\frac{2(x+5)}{(x+7)},\ x\neq -7,-4$$

e. $\dfrac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} + \frac{1}{x-2}}$

Method 1 : Add/Subtract num + den. by using common denominator

$\left(\dfrac{3}{(x-2)(x+2)}\cdot\dfrac{(x+2)}{(x+2)} - \dfrac{6}{x^2-4}\right) \div \left(\dfrac{3}{(x+2)}\cdot\dfrac{(x-2)}{(x-2)} + \dfrac{1}{(x-2)}\cdot\dfrac{(x+2)}{(x+2)}\right)$

$(x+2)(x-2)$

$\dfrac{3(x+2)-6}{(x+2)(x-2)} \div \dfrac{3(x-2)+1(x+2)}{(x+2)(x-2)}$

$\dfrac{3x+6-6}{(x+2)(x-2)} \div \dfrac{3x-6+x+2}{(x+2)(x-2)}$

$\dfrac{3x}{(x+2)(x-2)} \div \dfrac{4x-4}{(x+2)(x-2)}$ $\quad 4(x-1)$

$\dfrac{3x}{\cancel{(x+2)(x-2)}} \cdot \dfrac{\cancel{(x+2)(x-2)}}{4(x-1)} = \dfrac{3x}{4(x-1)}, x \neq 1, 2, -2$

Sec. 8.6 Solving Rational Equations

Problem 1:

a. $8\left(\frac{1}{4} - x\right) = \left(\frac{x}{8}\right)8$ LCD: 8

$2\cancel{8}\cdot\frac{1}{\cancel{4}} - 8x = \frac{x}{\cancel{8}}\cdot\cancel{8}$

$2 - 8x = x$

$+8x \quad +8x$

$\frac{2}{9} = \frac{9x}{9}$

$\frac{2}{9} = x$

b. $2y\left(\frac{2}{y} + \frac{1}{2}\right) = \left(\frac{5}{2y}\right)2y$ LCD: $2y$

$\cancel{2y}\cdot\frac{2}{\cancel{y}} + \cancel{2}y\cdot\frac{1}{\cancel{2}} = \frac{\cancel{2y}\cdot 5}{\cancel{2y}}$

$4 + y = 5$

$-4 \quad -4$

$y = 1$

C. $\frac{4(x+2)(2x+3)}{(x+2)} = \frac{5(x+2)(2x+3)}{(2x+3)}$ LCD: $(x+2)(2x+3)$

$x \neq -\frac{3}{2}$

$4(2x+3) = 5(x+2)$

$8x + 12 = 5x + 10$

$-5x \quad -5x$

$3x + 12 = 10$

$-12 \quad -12$

$\frac{3x}{3} = \frac{-2}{3}$

$x = \frac{-2}{3}$

d. $\left(\frac{1}{x^2-5x} + \frac{x-7}{x}\right) = \left(\frac{4}{x^2-5x}\right)^{x(x-5)}$ LCD: $x(x-5)$

$x(x-5)$ $x(x-5)$ x $x(x-5)$

$x(x-5)\cdot\frac{1}{x(x-5)} + x(x-5)\frac{(x-7)}{x} = x(x-5)\cdot\frac{4}{x(x-5)}$

$1 + (x-5)(x-7) = 4$

$1 + x^2 - 7x - 5x + 35 = 4$

$x^2 - 12x + 36 = 4$

$-4 \quad -4$

$x^2 - 12x + 32 = 0$ 4.8

$(x-4)(x-8) = 0$

$x = 4, 8$

e. $\dfrac{-(7x+3)}{x^2-8x+15} + \dfrac{-3x}{x-5} = \dfrac{1}{3-x}$

$-(x-3)(x-5)$ $\quad -(x-3)(x-5)$ $\quad -(x-3)(x-5)$

LCD: $-(x-3)(x-5)$

$-(7x+3) - 3x(x-3) = x-5$

$-7x-3-3x^2+9 = x-5$

$-3x^2-7x+6 = x-5$

$\quad -x+5 \quad -x+5$

$-3x^2-8x+11 = 0$

$\dfrac{-3x^2}{-1} - \dfrac{8x}{-1} + \dfrac{11}{-1} = \dfrac{0}{-1}$

$3x^2+8x-11 = 0$

$(3x+11)(x-1) = 0$

$x = 1, -\frac{11}{3}$

b. $\frac{5}{x} + \frac{x+1}{(x+2)} = \frac{2x+9}{(x+2)}$ LCD: $x(x+2)$

$\frac{5}{x} \cdot x(x+2) + \frac{(x+1)}{(x+2)} \cdot x(x+2) = \frac{(2x+9)}{(x+2)} \cdot x(x+2)$

$5(x+2) + (x+1)x = (2x+9)x$

$5x + 10 + x^2 + 1x = 2x^2 + 9x$

$x^2 + 6x + 10 = 2x^2 + 9x$

$-x^2 - 6x - 10$