

e. $y = x^4 + 3x^3 + x^2 - 12x - 20$

$\frac{P}{Q}$: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{r|rrrrr} & 4 & 3 & 2 & 1 & 0 \\ \hline 1 & 1 & 3 & 1 & -12 & -20 \\ & & 1 & 4 & 5 & -7 \\ \hline & 1 & 4 & 5 & -7 & | & X \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 3 & 1 & -12 & -20 \\ & & 2 & 10 & 22 & 20 \\ \hline -1 & 1 & 5 & 11 & 10 & 0 \\ & & -1 & -4 & -7 & \\ \hline & 1 & 4 & 7 & 3 & \end{array}$$

1 real \oplus zero
 $f(x) = x^4 - 3x^3 + x^2 + 12x - 20$
 3 or 1 \ominus real zero
 0 or 2 \oplus zeros

$$x = 2, -2, \frac{-3 \pm i\sqrt{11}}{2}$$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 11 & 10 \\ & & -2 & -6 & -10 \\ \hline & 1 & 3 & 5 & 0 \end{array}$$

$$x^2 + 3x + 5 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(5)}}{2 \cdot 1}$$

$$\frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm i\sqrt{11}}{2}$$

f. $y = x^4 + x^3 - 15x^2 - 16x - 16$ DRS:
 1 \oplus real root
 3 or 1 \ominus real root

$\frac{P}{Q}$: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} & 4 & 3 & 2 & 1 & 0 \\ \hline 1 & 1 & 1 & -15 & -16 & -16 \\ & & 1 & 2 & -13 & - \\ \hline & 1 & 2 & -13 & - & \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -15 & -16 & -16 \\ & & 2 & 6 & -18 & -6 \\ \hline & 1 & 3 & -9 & -33 & \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 1 & 1 & -15 & -16 & -16 \\ & & 4 & 20 & 20 & 16 \\ \hline -4 & 1 & 5 & 5 & 4 & 0 \\ & & -4 & -4 & -4 & \\ \hline & 1 & 1 & 1 & 0 & \end{array}$$

$$x = 4, -4, \frac{-1 \pm i\sqrt{3}}{2}$$

$$\begin{array}{r|rrrr} -4 & 1 & 5 & 5 & 4 & 0 \\ & & -4 & -4 & -4 & \\ \hline & 1 & 1 & 1 & 0 & \end{array}$$

$$x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2 \cdot 1}$$

$$\frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$