Sec. 5.1 Polynomial Functions

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$

(n → degree of the polynomial)

a_n: leading coefficient

a_0: Constant

Standard Form

$P(x) = 3x^5 - 2x^4 + 5x^2 - 3x + 7$ → Constant

$3x^5$: Quintic; $-2x^4$: Quartic; $0x^3$: Cubic; $5x^2$: Quadratic; $-3x$: Linear

Degree:
- term: sum of the variables' exponents
  Ex: $3xy^3z^7$ (11)
- polynomial: degree of the term with the highest degree Ex: $3x^5 - x^7 + 2x$ D: 7

Polynomials are closed under ⊕ ⊖ ⊗

* If you ⊕, ⊖, ⊗ polynomials, you get a polynomial.

Problem 1: Write in standard form and classify

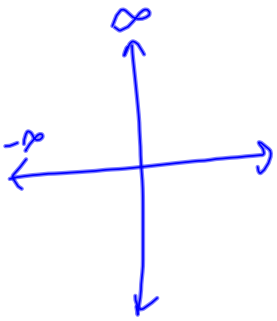
a. $3x^4 - x + 5x^4$
$5x^4 + 3x^3 - x$
Quartic
Trinomial
D: 4

b. $-3 - 4x^5 + 2x^2 + 10$
$-4x^5 + 2x^2 + 13$ $(13x^0)$
Quintic
Trinomial
D: 5

End Behavior

	Even Degree	Odd Degree
a ⊕	↑ ↑ Up Up	↓ ↑ Down Up
a ⊖	↓ ↓ Down Down	↑ ↓ Up Down

Problem 2: What is the end behavior of



a. $y = 3x^4 - 2x^3 + x - 1$

D: 4 → EVEN ↑↑ (+) up, up

as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to \infty$

b. $y = -4x^3 + 2x^2 + 7$

D: 3 ODD ↘↗ up, down

as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to -\infty$

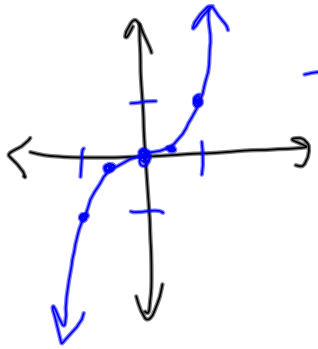
Turning Points: up to $n-1$

x^2 ∪ ∩ D: 2 TP: 1

x^5 D: 5 TP: 4

$y = x^5$ TP: 0

$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$



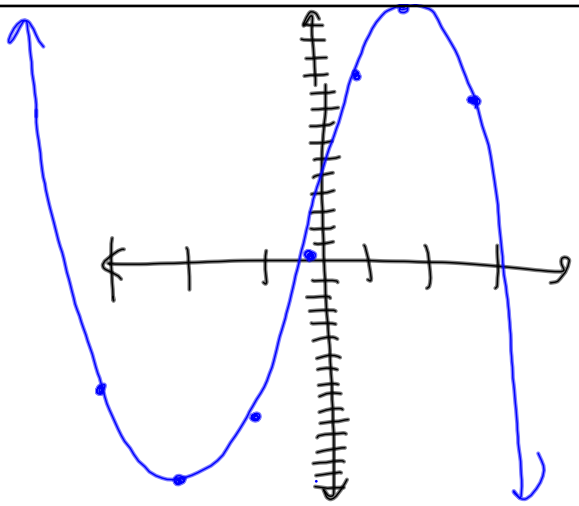
Graph:

$y = -x^3 + 12x$

EB: D: 3 ODD

(+) (−)

x	y	
0	0	
1	11	−1 + 12
−1	−11	1 − 12
2	16	−8 + 24
−2	−16	−(−8) − 24
3	9	−27 + 36
−3	−9	

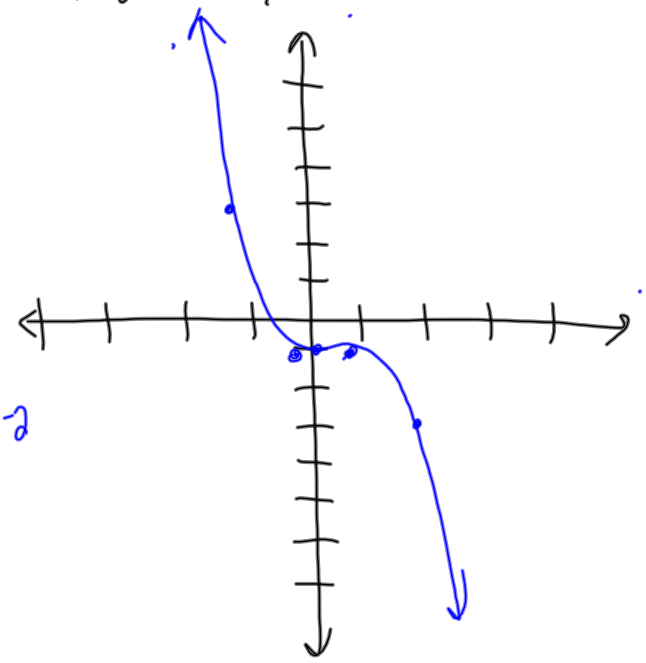


Graph $y = -x^3 + 2x^2 - x - 2$

EB: ODD

(+) (−)

x	y	
0	−2	
1	−2	−1+2−1−2
2	−4	−8+8−2−2
−1	2	1+2+1−2
$\frac{1}{2}$	$-2\frac{1}{8}$	$-\frac{1}{8} + \frac{1}{2} - \frac{1}{2} - 2$



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Problem 4: What is the degree of the polynomial function that generates the data shown?

a.

x	y
-3	23
-2	-16
-1	-15
0	-10
1	-13
2	-12
3	29

Differences

1st	2nd	3rd
-39	40	-36
1	4	-12
5	-8	12
-3	4	24
1	40	36
41		

-16-23

-15-(-16)

24

24

24

4th

Degree: 4

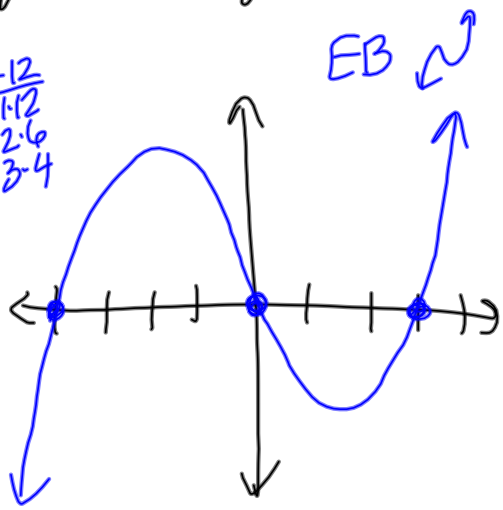
Sec. 5.2 Polynomials, Linear Factors, and Zeros

Problem 1: What is the factored form?

a. $x^3 + x^2 - 12x$

$x(x^2 + x - 12)$ $\begin{matrix} -12 \\ 1\cdot 12 \\ 2\cdot 6 \\ 3\cdot 4 \end{matrix}$

$x(x-3)(x+4)$



b. $x^3 - 36x$

$x(x^2 - 36)$

$x(x-6)(x+6)$

Roots, Zeros, and x-intercepts

$P(x) \rightarrow$ $x-b$ is a factor

* b is a zero of $P(x)$
* b is a root (or solution) of $P(x) = 0$.
* b is an x-intercept of the graph of $y = P(x)$

Problem 2:

What are the zeros? Graph.

a. $y = (x-3)(x+4)(x-1)$

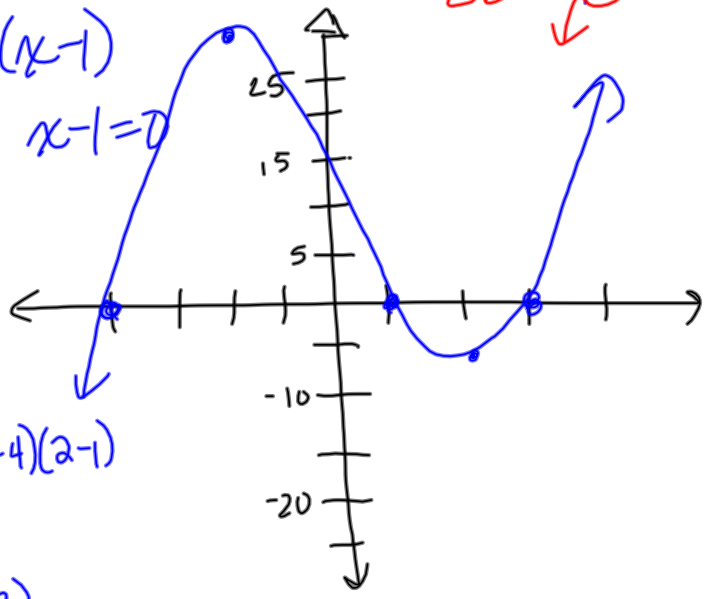
EB: x^3

$0 = (x-3)(x+4)(x-1)$

$x-3=0 \quad x+4=0 \quad x-1=0$

$x = 3, -4, 1$

x	y	
2	-6	$(2-3)(2+4)(2-1)$ $-1(6)(1)$
-2	30	$(-5)(2)(-3)$



Problem 3:

What is a cubic polynomial with zeros:

b → zero

x − b → factor

a. 1, −1, and 4

$P(x) = (x-1)(x+1)(x-4)$

$x^2 + 1x - 1x - 1$

$(x^2 - 1)(x-4)$

$P(x) = x^3 - 4x^2 - x + 4$

$x = 1$

$-1 \quad -1$

$(x-1) = 0$

① Write factors $(x-b_1)(x-b_2)\ldots$

② Multiply

b. 3, 3, −3

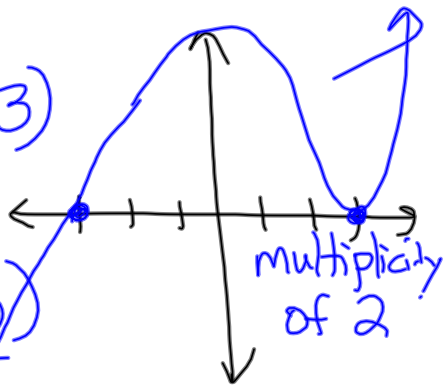
$P(x) = (x-3)(x-3)(x+3)$

$x^2 - 3x - 3x + 9$

$(x^2 - 6x + 9)(x+3)$

$x^3 - 6x^2 + 9x + 3x^2 - 18x + 27$

$P(x) = x^3 - 3x^2 - 9x + 27$



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Problem 4:

What are the zeros and multiplicities of

$f(x) = x^3 - 4x^2 + 4x$

$0 = x(x^2 - 4x + 4)$

$0 = x(x-2)(x-2)$

$x = 0, 2, 2$

$0 \rightarrow M \rightarrow 1$

$2 \rightarrow M \rightarrow 2$

$f(x) = x^3 = 0$

$x \cdot x \cdot x = 0$

$x = 0, x = 0, x = 0$

$0 \rightarrow$ Multiplicity of 3.