

Sec. 4.6 Solve Exponential and Logarithmic Equations

Recall:

$$\text{If } y = b^x, \text{ then } \log_b y = x$$

↑
exponent

$$b^{\log_b x} = x$$

$$\log_b b^x = x$$

$$3^{\log_3 9} = 9$$

↖ 3² →

$$\log_3 3^2 = 2$$

$$\log_5 125 = 3$$

5³
5²⁵
55

Properties:

$$\log_b m + \log_b n = \log_b m \cdot n$$

$$\log_b m - \log_b n = \log_b \frac{m}{n}$$

$$n \log_b m = \log_b m^n$$

* Change of Base

$$\log_a c = \frac{\log_b c}{\log_b a} = \frac{\log c}{\log a} = \frac{\ln c}{\ln a}$$

Solve the equation.

$$1. \quad 3^x = \left(\frac{1}{9}\right)^{x+3} \rightarrow \text{Bases need to be equal}$$

$$3^x = \left(\frac{1}{3^2}\right)^{x+3}$$

$$3^x = (3^{-2})^{(x+3)}$$

$$3^{\boxed{x}} = 3^{\boxed{-2x-6}}$$

$$\left(\log_3 3^x = \log_3 3^{(-2x-6)} \right)$$

$$x = -2x - 6$$

$$\begin{array}{r} +2x \quad +2x \\ \hline \end{array}$$

$$\frac{3x}{3} = \frac{-6}{3}$$

$$\boxed{x = -2}$$

$$b. \quad 9^x = 35 \rightarrow \begin{array}{l} \text{log form} \\ \log_9 35 = x \end{array}$$

$$\log_9 9^x = \log_9 35$$

$$x = \log_9 35 \text{ or } \frac{\log 35}{\log 9}$$

$$\approx 1.62$$

$$c. \textcircled{1} \quad \log_4 (\underline{2x+8}) = \log_4 (\underline{6x-12})$$

$$\textcircled{2} \quad \begin{array}{r} 2x + 8 = 6x - 12 \\ -2x \quad -2x \\ \hline \end{array}$$

$$\begin{array}{r} 8 = 4x - 12 \\ +12 \quad +12 \\ \hline \end{array}$$

$$\begin{array}{r} 20 = 4x \\ 4 \quad 4 \end{array}$$

$$\boxed{5 = x}$$

Check for
E.S.

$$d. \quad \log_7 (3x^{\oplus} - 2) = 2$$

$$\begin{array}{r} 3x - 2 = 49 \\ \hline +2 \quad +2 \\ \hline 3x = 51 \\ \frac{3x}{3} = \frac{51}{3} \\ \boxed{x = 17} \end{array} \quad * \text{Check}$$

e. $\log_6 3x + \log_6 (x-4) = 2$

Condense into one expression \rightarrow

$$\log_6 m + \log_6 n = \log_6 m \cdot n$$

$6^2 = 3x(x-4)$ \leftarrow

$$\log_6 3x(x-4) = 2$$

$$3x(x-4) = 36$$

$$\frac{3x^2}{3} - \frac{12x}{3} = \frac{36}{3}$$

$$x^2 - 4x = 12$$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$

$$x = -2, 6$$

