# Sec. 4.2 Standard Form of a Quadratic Function

Standard Form: $f(x) = ax^2 + bx + c$ (where $a \neq 0$)

$y = a(x-h)^2 + k$

Problem 1:

What are the vertex, axis of symmetry, maximum or minimum value, and range of:

a. $y = -x^2 + 6x + 3 = -(3)^2 + 6(3) + 3$ *

$a = -1 \quad b = 6 \quad c = 3$

$-9 + 18 + 3$

$9 + 3$

$12$

$\boxed{\frac{-b}{2a}}$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $(3, 12)$

axis of symmetry: $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

$\boxed{x = 3}$

maximum: 12

range: $y \leq 12$

If $a > 0$, ∪, minimum: $f\left(\frac{-b}{2a}\right)$

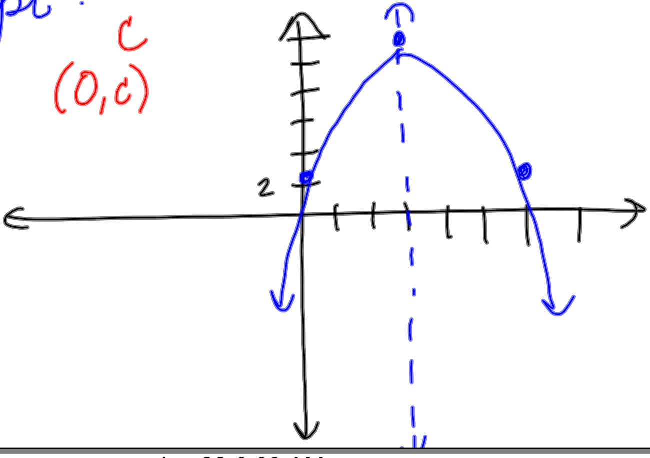
range $y \geq f\left(\frac{-b}{2a}\right)$

If $a < 0$, ∩, maximum: $f\left(\frac{-b}{2a}\right)$

range: $y \leq f\left(\frac{-b}{2a}\right)$

y-intercept: $c$

$(0, c)$




| $x$ | $y$ |
|---|---|
| 0 | $c$ |
| y-interc | |
| 0 | 3 |

b. $y = 4x^2 - 16x \boxed{+10}$ y-intercept

aos: $x = \frac{-b}{2a} = \frac{16}{2\cdot 4} = \frac{16}{8} = 2$ $\boxed{x=2}$

v: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $y = 4(2)^2 - 16(2) + 10$

$y = 4\cdot 4 - 32 + 10$

$16 - 32 + 10$

$-16 + 10$

$-6$

$(2, -6)$

max/min: −6

range: $y \geq -6$

y-intercept: 10
$(x=0)$

$y = 4x^2 - 16x + 10$

| $x$ | $y$ |
|---|---|
| 0 | 10 |
| 1 | −2 |

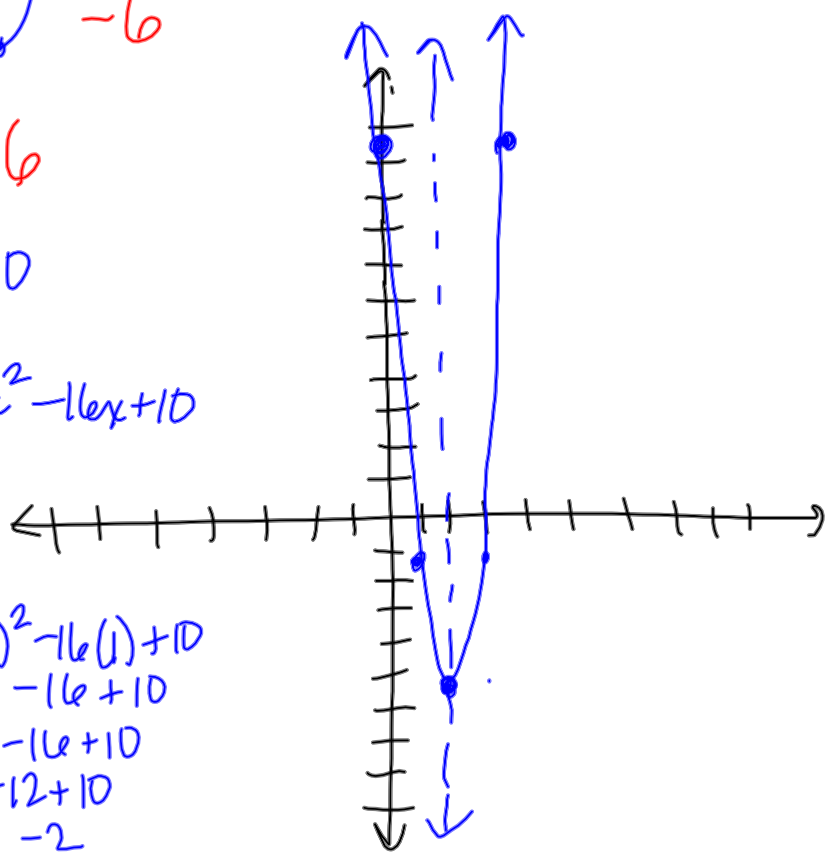
$y = 4(1)^2 - 16(1) + 10$

$4\cdot 1 - 16 + 10$

$4 - 16 + 10$

$-12 + 10$

$-2$



Problem 2:

What is the vertex form of

a. $y = 2x^2 - 3x + 2$

$a = 2 \quad b = -3 \quad c = 2$

$y = a(x-h)^2 + k$

$y = 2\left(x - \frac{3}{4}\right)^2 + \frac{7}{8}$

$y = 2(x - 0.75)^2 + 0.875$

$x_v = \frac{-b}{2a} = \frac{3}{2(2)} = \frac{3}{4}$ $v: \left(\frac{3}{4}, \frac{7}{8}\right)$

$(0.75, 0.875)$

$y_v = 2\left(\frac{3}{4}\right)^2 - \frac{3}{1}\left(\frac{3}{4}\right) + 2$

$\frac{3}{4} \cdot \frac{3}{4}$

$2\left(\frac{9}{16}\right) - \frac{9}{4} + 2$

$\frac{9}{8} - \frac{9}{4} \cdot \frac{2}{2} + \frac{2}{1} \cdot \frac{8}{8}$

$\frac{9}{8} - \frac{18}{8} + \frac{16}{8}$

$-\frac{9}{8} + \frac{16}{8}$

$\boxed{\frac{7}{8}}$

$$f(x) = \sqrt{3x-4}$$

$$g(x) = x^2 + 5$$

$$f \circ g(x) = f(g(x))$$

$$f(x^2+5) = \sqrt{3(x^2+5)-4} = \sqrt{3x^2+15-4}$$

$$= \sqrt{(3x^2+11)}$$

$$f(x) = \sqrt{3x-4}$$

$$g \circ f(x) = g(f(x)) = g(\sqrt{3x-4})$$

$$= \left(\sqrt{3x-4}\right)^2 + 5$$

$$3x - 4 + 5$$

$$= 3x + 1$$

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b. $y = -x^2 + 4x - 5$

$a = -1 \quad b = 4 \quad c = -5$

$y = a(x-h)^2 + k$

$\frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$y = -1(x-2)^2 - 1$

$\boxed{y = -(x-2)^2 - 1}$

$y = -(2)^2 + 4(2) - 5$

$-4 + 8 - 5$

$4 - 5$

$-1$

v: $(2, -1)$

c. $y = -2x^2 + 8x + 3$

$a = -2 \quad b = 8 \quad c = 3$

$y = a(x-h)^2 + k$

$\boxed{y = -2(x-2)^2 + 11}$

v: $(2, 11)$

$h \quad k$

$\frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = \boxed{2}$

$y = -2(2)^2 + 8(2) + 3$

$-2 \cdot 4 + 16 + 3$

$-8 + 16 + 3$

$8 + 3$

$11$

v: $(2, 11)$

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Problem 3:

A model for performance of a stock is $P=-3d^2+50d$, where d represents the days of trading and P is the price per share. What is the maximum price per share of the stock?

"k"

$y_v$

$a=-3 \quad b=50 \quad c=0$

$$\frac{-b}{2a} = \frac{-50}{2(-3)} = \frac{-50}{-6} = \frac{25}{3} \qquad \left(\frac{25}{3}, \frac{625}{3}\right)$$

$$P = -3\left(\frac{25}{3}\right)^2 + 50\left(\frac{25}{3}\right) \qquad \left(8\tfrac{1}{3}, 208\tfrac{1}{3}\right)$$

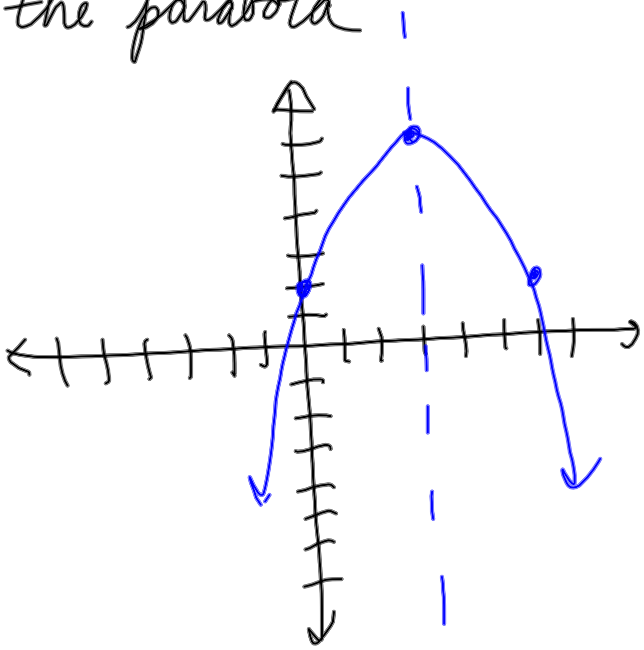
$$-\cancel{3}^{1}\left(\frac{625}{\cancel{9}_3}\right) + \frac{1250}{3}$$

$$-\frac{625}{3} + \frac{1250}{3}$$

$$\frac{625}{3} = 208\tfrac{1}{3} \ \boxed{\$208.33}$$

vertex: (3, 6)   y-int: 2

Graph the parabola



Find the unknown coefficients.

$y = x^2 + bx + c$  v: $(3, -4)$ $\quad x_v \quad y_v$

$a = 1$  $\boxed{b = -6}$  $\boxed{c = 5}$

$$\frac{-b}{2a} = x_v$$

$$\frac{-b}{2(1)} = 3$$

$$2 \cdot \frac{-b}{2} = 3 \cdot 2$$

$$\frac{-b}{-1} = \frac{6}{-1}$$

$$b = -6$$

$$y = x^2 + bx + c$$

$$-4 = (3)^2 + (-6)(3) + c$$

$$-4 = 9 - 18 + c$$

$$-4 = -9 + c$$

$$+9 \quad +9$$

$$5 = c$$

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$y = x^2 + bx + c$  v: $(\overset{h}{3}, \overset{k}{-4})$

$a = 1$  $\boxed{b =\ }$  $\boxed{c =\ }$

$$y = a(x-h)^2 + k$$

$$y = 1(x-3)^2 - 4$$

$$y = (x-3)(x-3) - 4$$

$$y = x^2 - 3x - 3x + 9 - 4$$

$$y = x^2 - 6x + 5$$

$\boxed{b = -6}$  $\boxed{c = 5}$

Find the missing coefficients

$y = ax^2 + 10x + c$ v: $(-5, -27)$ $x_v$ $y_v$

$\boxed{a = 1}$ $b = 10$ $\boxed{c = -2}$

$\frac{-b}{2a} = x_v$

$2a \cdot \frac{-10}{2a} = -5 \cdot 2a$

$\frac{-10}{-10} = \frac{-10a}{-10}$

$1 = a$

$y = ax^2 + 10x + c$

$-27 = 1(-5)^2 + 10(-5) + c$

$-27 = 25 - 50 + c$

$-27 = -25 + c$

$+25 \quad +25$

$-2 = c$

$y = ax^2 + 10x + c$ $(-5, -27)$

$\boxed{a = 1}$ $b = 10$ $\boxed{c = -2}$

$\frac{-b}{2a} = x_v$

$2a \cdot \frac{-10}{2a} = -5 \cdot 2a$

$\frac{-10}{-10} = \frac{-10a}{-10}$

$1 = a$

$y = a(x-h)^2 + k$

$y = a(x+5)^2 - 27$

$y = 1(x+5)^2 - 27$

$y = (x+5)(x+5) - 27$

$y = x^2 + 5x + 5x + 25 - 27$

$y = x^2 + 10x - 2$

$a = 1$ $c = -2$

Find the y-intercept Make $x=0$.

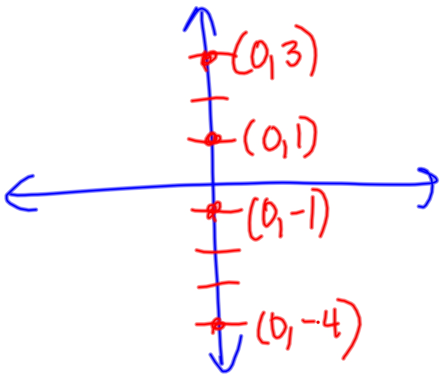
$$y=(x-1)^2+2$$

$$y=(0-1)^2+2$$

$$y=(-1)^2+2$$

$$y=1+2$$

$$y=3$$




Change to standard form.

$$y=(x-1)^2+2$$

$$y=(x-1)(x-1)+2$$

$$y=x^2-1x-1x+1+2$$

$$\boxed{y=x^2-2x+3}$$

y-int: 3