

Sec. 9.4 Factoring to Solve Quadratic Equations

Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$ for all real numbers a and b .

Problem 1:

What is the solution of the equation

a. $(2x+3)(x-1) = 0$?

$$\begin{array}{r} 2x+3=0 \\ -3 \quad -3 \\ \hline 2x = -3 \\ \frac{2x}{2} = \frac{-3}{2} \\ x = -\frac{3}{2} \end{array} \qquad \begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline x = 1 \end{array}$$

b. $(x-1)(x+5) = 0$

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline x = 1 \end{array} \qquad \begin{array}{r} x+5=0 \\ -5 \quad -5 \\ \hline x = 5 \end{array}$$

c. $(7m-2)(5m+4) = 0$

$$\begin{array}{r} 7m-2=0 \\ +2 \quad +2 \\ \hline 7m = 2 \\ \frac{7m}{7} = \frac{2}{7} \\ m = \frac{2}{7} \end{array} \qquad \begin{array}{r} 5m+4=0 \\ -4 \quad -4 \\ \hline 5m = -4 \\ \frac{5m}{5} = \frac{-4}{5} \\ m = -\frac{4}{5} \end{array}$$

Problem 2: Solve

$$a. x^2 - x - 12 = 0 \quad \begin{array}{r} -12 \\ 1 \cdot 12 \\ 2 \cdot 6 \\ 3 \cdot 4 \end{array}$$

$$(x+3)(x-4) = 0$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline x=-3 \end{array} \quad \begin{array}{r} x-4=0 \\ +4 \quad +4 \\ \hline x=4 \end{array}$$

$$b. m^2 - 5m - 14 = 0 \quad \begin{array}{r} -14 \\ 1 \cdot 14 \\ 2 \cdot 7 \end{array}$$

$$(m+2)(m-7) = 0$$

$$\begin{array}{r} m+2=0 \\ -2 \quad -2 \\ \hline m=-2 \end{array} \quad \begin{array}{r} m-7=0 \\ +7 \quad +7 \\ \hline m=7 \end{array}$$

$$c. p^2 + p - 20 = 0 \quad \begin{array}{r} -20 \\ 1 \cdot 20 \\ 2 \cdot 10 \\ -4 \cdot 5 \end{array}$$

$$(p-4)(p+5) = 0$$

$$\begin{array}{r} p-4=0 \\ +4 \quad +4 \\ \hline p=4 \end{array} \quad \begin{array}{r} p+5=0 \\ -5 \quad -5 \\ \hline p=-5 \end{array}$$

$$d. 2a^2 - 15a + 18 = 0 \quad \begin{array}{r} 36 \\ 1 \cdot 36 \\ 2 \cdot 18 \\ -3 \cdot 12 \\ \hline 4 \cdot 9 \end{array}$$

$$2a^2 - 3a - 12a + 18 = 0$$

$$a(2a-3) - 6(2a-3) = 0$$

$$(2a-3)(a-6) = 0$$

$$\begin{array}{r} 2a-3=0 \\ +3 \quad +3 \\ \hline 2a=3 \\ \frac{2a}{2} = \frac{3}{2} \\ a = \frac{3}{2} \end{array}$$

$$\begin{array}{r} a-6=0 \\ +6 \quad +6 \\ \hline a=6 \end{array}$$

$$a = \frac{3}{2}$$

Problem. 3: Solve.

$$\text{a. } \begin{array}{r} 3x^2 + 13x = -4 \\ \quad \quad +4 \quad +4 \\ \hline \end{array}$$

$$\begin{array}{l} 3x^2 + 13x + 4 = 0 \\ \underline{3x^2 + 1x + 12x + 4 = 0} \end{array}$$

$$x(3x+1) + 4(3x+1) = 0$$

$$\text{b. } \begin{array}{r} x^2 + 14x = -49 \\ \quad \quad +49 \quad +49 \\ \hline \end{array}$$

$$x^2 + 14x + 49 = 0$$

$$(x+7)^2 = 0 \quad (x+7)(x+7) = 0$$

$$\begin{array}{r} x+7=0 \\ \underline{-7 \quad -7} \end{array}$$

$$x = -7$$

$$\text{c. } \begin{array}{r} 4x^2 - 21x = 18 \\ \quad \quad -18 \quad -18 \\ \hline \end{array}$$

$$4x^2 - 21x - 18 = 0$$

$$\underline{4x^2 + 3x - 24x - 18 = 0}$$

$$x(4x+3) - 6(4x+3) = 0$$

$$(x-6)(4x+3) = 0$$

$$\begin{array}{r} x-6=0 \\ \underline{+6 \quad +6} \end{array}$$

$$x = 6$$

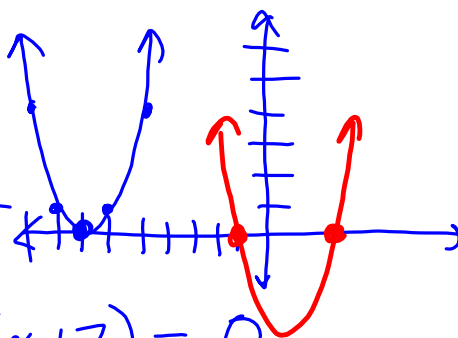
$$\begin{array}{r} 4x+3=0 \\ \underline{-3 \quad -3} \end{array}$$

$$\frac{4x}{4} = \frac{-3}{4}$$

$$x = -\frac{3}{4}$$

$$\begin{array}{r} 12 \\ \hline 1 \cdot 12 \\ 2 \cdot 6 \\ 3 \cdot 4 \end{array}$$

$$(3x+1)$$



$$\begin{array}{r} -72 \\ \hline 1 \cdot 72 \\ 2 \cdot 36 \\ \hline 3 \cdot 24 \\ 4 \cdot 18 \\ 6 \cdot 12 \\ 8 \cdot 9 \end{array}$$

Problem 4:

Jason has a patio of uniform width around the perimeter of his rectangular pool. The pool measures 22 ft by 12 ft. If the area of the pool and the patio is 504 ft^2 , what is the width of the patio?

$$A = l \cdot w$$

$$504 = (2x + 22)(2x + 12)$$

$$504 = 4x^2 + 24x + 44x + 264$$

$$\frac{0}{4} = \frac{4x^2}{4} + \frac{68x}{4} - \frac{240}{4}$$

$$0 = x^2 + 17x - 60$$

$$0 = (x - 3)(x + 20)$$

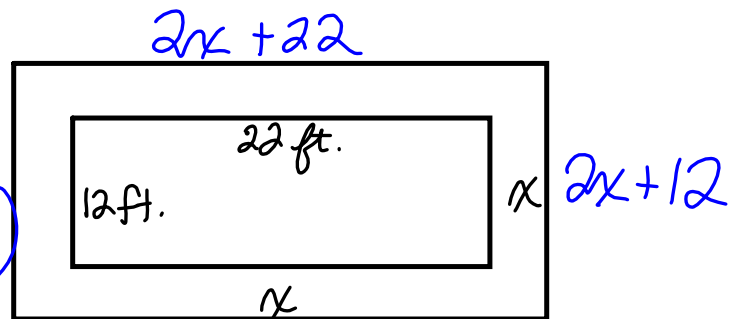
$$\begin{array}{r} x - 3 = 0 \\ +3 \quad +3 \\ \hline \end{array}$$

$$x = 3$$

$$\begin{array}{r} x + 20 = 0 \\ -20 \quad -20 \\ \hline \end{array}$$

$$x = -20$$

$$\boxed{3 \text{ ft}}$$



$$\begin{array}{r} 12 \\ 22 \\ \hline 24 \\ 24 \\ \hline 264 \end{array}$$

$$\begin{array}{r} -60 \\ 1 \cdot 60 \\ 2 \cdot 30 \\ 3 \cdot 20 \end{array}$$