Sec. 4.7 Arithmetic Sequences

Vocabulary

- sequence: an ordered list of numbers, that often form a pattern
- term of a sequence: each number in the list
- arithmetic sequence: a sequence formed by adding a fixed number to each previous term to find the next term
- common difference: the fixed number; the difference between consecutive terms in an arithmetic sequence

ex: 5, 3, 1, −1, . . .

−2 −2 −2 : common difference : −2

3−5, 1−3, −1−1

- recursive formula: a function rule that relates each term of a sequence after the first to the one before it

$$A(n) = A(n-1) + d$$

(n^{th} term) = (value of the previous term) + (common difference)

- explicit formula: a function rule that relates each term of a sequence to the term number.

$$A(n) = A(1) + (n-1)d$$

$A(n)$: n^{th} term; $A(1)$: first term; $(n-1)$: term number − 1; d: common difference

Problem 1: Describe the pattern and find the next two terms of the sequence.

a. 4, 6, 8, 10, ... add 2 to the previous term
12, 14

b. 1, 3, 9, 27, ... multiply the previous term by 3
81, 243

Problem 2: Tell whether the sequence is arithmetic. If it is, what is the common difference?

a. 7, 11, 16, 22, ... not arithmetic
4 5 6

b. 3, 9, 15, 21, ... arithmetic CD: 6
6 6 6

Problem 3: Write a recursive formula for the arithmetic sequence

① ② ③ ④
25, 31, 37, 43, ...

6 6 6 CD: 6

What is the 7th term?

$A(n) = A(n-1) + d$

next term previous term CD

$A(n) = A(n-1) + 6$

$A(5) = A(4) + 6 = 43 + 6 = 49$

$A(6) = A(5) + 6 = 49 + 6 = 55$

$A(7) = A(6) + 6 = 55 + 6 = \boxed{61}$

Problem 4: An arithmetic sequence is represented by the recursive formula $A(n) = A(n-1) + 15$. If the first term of the sequence is $\boxed{42} = A(1)$, write the explicit formula.

$A(n) = A(1) + (n-1)d$

$A(n) = 42 + (n-1)15$

Problem 5: An arithmetic sequence is represented by the explicit formula $A(n) = 8 + (n-1)(11)$. What is the recursive formula?

($A(1) = 8$, $d = 11$)

$A(n) = A(n-1) + d$

$A(n) = A(n-1) + 11$

Additional Examples

a. $A(n) = 15 + (n-1)(-7)$

Write the recursive formula.

$A(n) = A(n-1) + d$

$A(n) = A(n-1) - 7$

b. Write the recursive and explicit formula for the following arithmetic sequence. Find the 13th term.

26, 22, 18, 14, ...

$A(1) = 26$ -4 -4 -4 $d = -4$

$A(n) = A(n-1) + d$

$\boxed{A(n) = A(n-1) - 4}$ RF

$A(n) = A(1) + (n-1)d$

$\boxed{A(n) = 26 + (n-1)(-4)}$ EF

$n = 13$

$A(13) = 26 + (13-1)(-4)$

$A(13) = 26 + 12(-4)$

$= 26 - 48$

$A(13) = \boxed{-22}$