Sec. 12.6 Permutations and Combinations

Vocabulary

Multiplication Counting Principle

If there are m ways to make a first selection and n ways to make a second selection, there are $m \cdot n$ ways to make the two selections

Ex: 3 breads, 4 meats, 2 cheeses

$3 \cdot 4 \cdot 2 = 24$ choices

-permutation: an arrangement of objects in a specific order

ABC
ACB
BAC
BCA
CAB
CBA

$${}_nP_r = \frac{n!}{(n-r)!}$$

n → objects
r → how many arranging

$${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3 \cdot 2 \cdot 1}{0!} = \frac{6}{1} = 6$$

* $n! = n(n-1)(n-2)\cdots(1)$

n factorial

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

* $0! = 1$

- Combination : a selection of objects without regard to order

$$_nC_r = \frac{n!}{r(n-r)!}$$

$$_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!}$$

$$= \frac{5\cdot \cancel{4}^{2}\cdot \cancel{3\cdot 2\cdot 1}}{\cancel{2}\cdot 1\;\cancel{3\cdot 2\cdot 1}} = 10$$

$$\frac{5\cdot 4}{2\cdot 1}$$

Problem 1: A car manufacturer offers 4 different kinds of interiors and 9 different color options. How many different cars are possible?

$$4\cdot 9 = 36$$

Problem 2: How many different ways can 8 players finish a race if there are no ties?

$$8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$$

n objects arranged r at a time :

$$_nP_r = {}_8P_8 = \frac{n!}{(n-r)!}$$

$$= \frac{8!}{\underset{(0)!}{(8-8)!}} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{1} =$$

$$= 40{,}320$$

b. You have 10 books. In how many orders can you read 4 of them?

$$10\cdot 9\cdot 8\cdot 7 = \boxed{5040}$$

$$_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

Problem 3: There are 6 students running for 3 different student council positions: president, vice president, and treasurer. How many different ways can the three offices be filled?

$$6 \cdot 5 \cdot 4 = 120$$

$${}_6P_3 = \frac{6!}{\underset{3!}{(6-3)!}} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1}} = 120$$

Problem 4: Ten employees are eligible for 4 promotions. How many different ways can the promotions be earned if the order is not important?

*combination: selection WITHOUT regard to order

$${}_nC_r = \frac{n!}{\boxed{r!}(n-r)!}$$

$${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\,6!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= \boxed{210}$$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1}$$

b. $${}_7C_3 = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}$$

$${}_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2} \cdot 1 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= \boxed{35}$$

$$\frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}$$

1·2·3
2·1·3