Sec. 11.1 Simplifying Rational Expressions

rational expression: $\frac{\text{polynomial}}{\text{polynomial}}$

excluded value: a value for which a rational expression is undefined – when the denominator is zero

ex: $\frac{x+2}{x-3}$ $\quad x-3=0$, so $x \neq 3$

$x=3$

excluded

Problem 1: What is the simplified form of the expression? State any excluded values.

factor → e.v. → cancel

a. $\frac{(x+3)}{(2x+6)} = \frac{1}{\cancel{(x+3)}} \Big/ \frac{2(\cancel{x+3})}{1} = \frac{1}{2}$, $x \neq -3$

$x+3=0$

$-3 \quad -3$

$x=-3$

b. $\frac{5x+10}{x^2-x-6}$ $\quad 1 \cdot 6$, $2+3$

$= \frac{5(\cancel{x+2})}{1} \Big/ \frac{(\cancel{x+2})(x-3)}{1} = \frac{5}{x-3}$, $x \neq -2, 3$

$x+2=0$ $\quad -2 \quad -2$ $\quad x=-2$

$x-3=0$ $\quad +3 \quad +3$ $\quad x=3$

c. $\frac{x^2-9}{-x^2+2x+3}$ $\quad 0x$, $1 \cdot 9$, $-3 \cdot 3$

$= \frac{1}{(\cancel{x-3})(x+3)} \Big/ \frac{+1(x+3)}{-1(x+1)(\cancel{x-3})}$

$-1(x^2-2x-3)$

$1+3$

$= -\frac{x+3}{x+1}$ $\quad x \neq -1, 3$

Sec. 11.2 Multiplying and Dividing Rational Expressions

divide: multiply by the reciprocal

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Problem 1: What is the product?

a. $\frac{3}{x} \cdot \frac{-2}{x} = \frac{-6}{x^2}$, $x = 0$

b. $\frac{x}{x+3} \cdot \frac{x-2}{x+5} = \frac{x(x-2)}{(x+3)(x+5)}$, $x \neq -3, -5$

OR

$x(x+5)+3(x+5)$

$x^2+5x+3x+15 = \frac{x^2-2x}{x^2+8x+15}$, $x \neq -3, -5$

c. $\frac{x+1}{\frac{4x}{4}-\frac{8}{4}} \cdot \frac{x-2}{x^2+4x+3} = \frac{(x+1)(x-2)}{4(x-2)(x+1)(x+3)}$

$4(x-2)(x+1)(x+3)$

$\frac{1}{4(x+3)}$, $x = 2, -1, -3$

d. $\dfrac{3x+4}{\frac{3x}{3}-\frac{9}{3}} \cdot (x^2+5x-24)$ $\quad -24 = 1\cdot 24,\ 2\cdot 12,\ -3\cdot 8,\ 4\cdot 6$

$\dfrac{(3x+4)(x-3)(x+8)}{3(x-3)} = \dfrac{1(3x+4)(x+8)}{3}$ OR $\dfrac{1}{3}(3x+4)(x+8)$ $\quad x\neq 3$

$x\neq 3$

Problem 2: What is the quotient?

a. $\dfrac{x^2-36}{9x-18} \div \dfrac{x+6=0}{x^2+5x-14}$ $\quad 6\cdot 6,\ 14 = 1\cdot 14,\ 2\cdot 7$

$= \dfrac{x^2-36}{9x-18} \cdot \dfrac{x^2+5x-14}{x+6} = \dfrac{(x+6)(x-6)(x-2)(x+7)}{9(x-2)(x+6)}$

$\dfrac{(x^2-36)(x^2+5x-14)}{(9x-18)(x+6)}$

$\dfrac{(x-6)(x+7)}{9}$ $\quad x\neq 2,-7,-6$

b. $\dfrac{8x^3-8x}{4x} \div (x^2+5x+4)$

$\dfrac{8x^3-8x}{4x} \cdot \dfrac{1}{(x^2+5x+4)} = \dfrac{8x(x^2-1)}{4x(x+1)(x+4)} = \dfrac{8x(x+1)(x-1)}{4x(x+1)(x+4)}$

$x\neq 0,-1,-4$

$\dfrac{2(x-1)}{(x+4)}$, $x\neq 0,-1,-4$

c. $\dfrac{\frac{1}{x+4}}{\frac{x-3}{x^2-16}}$

$\dfrac{1}{x+4} \div \dfrac{x-3}{x^2-16}$

$\dfrac{1}{(x+4)} \cdot \dfrac{(x^2-16)}{(x-3)}$

$\dfrac{(x+4)(x-4)}{(x+4)(x-3)}$

$\dfrac{x-4}{x-3}$ $\quad x=-4,3,4$

Sec. 10.3 Operations With Radical Expressions

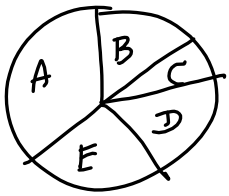
like radicals: have same radicand

ex: $3\sqrt{7} + 5\sqrt{7} = (3+5)\sqrt{7} = 8\sqrt{7}$

unlike radicals: $4\sqrt{3} + 2\sqrt{2}$ → cannot simplify

Simplify

Sec. 11.2 Probability



experimental probability

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

sample space: the set of all possible outcomes to an experiment or activity

equally likely outcome: each outcome in a sample space has the same chance of occurring

theoretical probability: If a sample space has n equally likely outcomes and an event A occurs in m of these outcomes, then the theoretical probability of event A is

$$P(A) = \frac{m}{n}$$

Problem 1: You observe 119 animals at a zoo, 19 of them have wings. What is the experimental probability that an animal at the zoo has wings?

$$P(\text{winged animal}) = \frac{19}{119} \approx 0.16$$

Problem 2: What is the theoretical probability of each event?

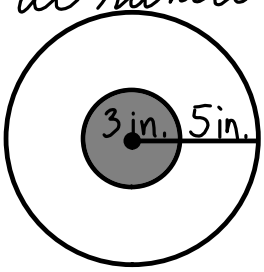
a. getting a number less than 3 on one roll of a fair number cube

b. getting a sum that is a multiple of 4 on one roll of two fair number cubes

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Problem 3: What is the theoretical probability of being dealt exactly three 8's in a 5-card hand from a standard 52-card deck?

Problem 4: A carnival game consists of throwing darts at a circular board as shown. What is the geometric probability that a dart thrown at random will hit the shaded circle?



Sec. 11.3 Probability of Multiple Events

dependent events : when the occurrence of one event affects how a second event can occur.

independent events : one event does not affect another

Probability of A and B

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$

mutually exclusive events : cannot happen at the same time

Note: If A and B are mutually exclusive then $P(A \text{ and } B) = 0$.

Probability of A or B

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$

Problem 1: Is each pair of events dependent or independent?

a. Flip a coin. Then roll a number cube.

b. Choose a marble from a bag. Keep the marble and then choose another marble from the same bag.

Problem 2: What is the probability of rolling a 6 on a fair number cube and flipping a coin and getting tails?

Problem 3: You select one card from a standard 52-card deck. Are the events mutually exclusive? Explain.

a. choosing a red card or an even numbered card

b. choosing a red card or a black card

Problem 4: Students choose one elective each school year.