Sec. 7.8 Geometric Sequences

$a, \ a \cdot r, \ ar \cdot r, \ ar^2 \cdot r$

$a, \ ar, \ ar^2, \ ar^3, \ ar^4, \ \ldots$

a → starting value

$r \rightarrow$ common ratio

Recursive definition

$a_1 = a$ Initial condition

$a_n = a_{n-1} \cdot r$ Recursive formula
for $n \geq 2$

Explicit definition

$a_n = a_1 r^{n-1}$, for $n \geq 1$

Problem 1:

Which are geometric sequences? If not geometric, is it arithmetic?

a. $3, 6, 12, 24, 48, \ldots \rightarrow \frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24}$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

$r = 2$, yes

Common ratio? yes, $r = 2$

b. $3, 6, 9, 12, 15, \ldots$ $\quad \frac{6}{3} = \frac{9}{6} = \frac{12}{9}$

$\cancel{\times 2} \quad \cancel{\times 2}$ $\quad 2 \neq \frac{3}{2}$

$+3 \quad +3 \quad +3 \quad +3 \longrightarrow$ arithmetic.

not geometric

common difference

$6-3 = 9-6 = 12-9 = 15-12 \quad d = 3$

c. $\frac{1}{3} \cdot \frac{1}{3} \quad \frac{1}{9} \cdot \frac{1}{3} \quad \frac{1}{27} \cdot \frac{1}{3} \quad \frac{1}{81}, \ldots$

yes, $r = \frac{1}{3}$

d. $4, 7, 11, 16, 22, \ldots$ $\quad \frac{7}{4} = \frac{11}{7} = \frac{16}{11}$

$7-4 = 11-7 = 16-11$

$3 \quad 4 \quad 5$

neither

Problem 2

Find the recursive formula and the explicit formula.

$a_4 = 16$ $a_5 = 16 \cdot 2$

a. 2, 4, 8, 16, ... $r = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$

$a_1 = 2$ $r = 2$

R: $a_1 = 2$; $a_n = a_{n-1} \cdot 2$ $\boxed{a_1 = a;\ a_n = a_{n-1} \cdot r}$

a_1 $a_2 = a_1 \cdot 2$

$2 \cdot 2$

$2,$ $4 \cdot 2;$ $8, 16$

$a_3 = a_2 \cdot 2$

$a_4 = a_3 \cdot 2$

8

E: $a_n = 2 \cdot 2^{n-1}$ $\boxed{a_n = a_1 r^{n-1}}$

b. 40, 20, 10, 5, ...

$\frac{40}{2}$

$a_1 = 40$ $r = \frac{1}{2}$

R: $a_1 = 40$; $a_n = a_{n-1} \cdot \frac{1}{2}$

E: $a_n = 40 \cdot \left(\frac{1}{2}\right)^{n-1}$

c. 5, 15, 45, 135, ...

$a_1 = 5$ $r = 3$

$a_7 \rightarrow$ the seventh term

R: $a_1 = 5$; $a_n = a_{n-1} \cdot 3$

E: $a_n = 5(3)^{n-1}$

$5 \cdot 3^{n-1}$

$a_n \rightarrow n^{th}$ term

$a_{n-1} \rightarrow$ term before the n^{th} term

Recursive formula

$-\frac{1}{50}, \quad \frac{1}{25}, \quad -\frac{2}{25}, \quad \frac{4}{25}, \ldots$ $\quad \cdot -2 \quad -2 \quad \cdot -2$

$a_1 = -\frac{1}{50}; \quad a_n = a_{n-1}(-2)$

$\dfrac{\frac{1}{25}}{-\frac{1}{50}} \quad \frac{1}{25} \cdot \frac{-50}{1} = -\frac{50}{25} = -2$

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