Sec. 7.5 Rational Exponents

Ex: $x^{\frac{1}{3}} = \sqrt[3]{x}$

$27^{\frac{1}{3}} = \sqrt[3]{27}$

$(3 \cdot 3 \cdot 3)^{\frac{1}{3}} = 3$

$25^{\frac{1}{2}} \rightarrow \sqrt{25}$

$(5 \cdot 5)^{\frac{1}{2}} = 5$

Simplify

a. $\sqrt[3]{125} = 5$

5 | 25

5 | 5

b. $\sqrt[4]{16} = 2$

4 4

2 | 2 | 2 | 2

* note: $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Ex: $x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{1}{3}} = \sqrt[3]{x}$

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Ex: a. $\sqrt[4]{81} = 81^{\frac{1}{4}} = (3\cdot3\cdot3\cdot3)^{\frac{1}{4}} = 3$

$81 = 9 \cdot 9$

$3\ 3\ 3\ 3$

b. $\sqrt[3]{1728} = (1728)^{\frac{1}{3}} = (2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot3)^{\frac{1}{3}}$

$1728 = 2 \cdot 864$, $864 = 2 \cdot 432$, $432 = 2 \cdot 216$, $216 = 2 \cdot 108$, $108 = 2 \cdot 54$, $54 = 9 \cdot 6$, $6 = 2 \cdot 3$

$= 2\cdot2\cdot3 = 12$

c. $100x^{\frac{1}{2}} = 100\sqrt{x}$

d. $(64x)^{\frac{1}{2}} = (64)^{\frac{1}{2}} x^{\frac{1}{2}} = \boxed{8\sqrt{x}}$

$\sqrt{64}$

e. $(54x)^{\frac{2}{3}} = 54^{\frac{2}{3}} x^{\frac{2}{3}}$

$54 = 9 \cdot 6$, $9 = 3 \cdot 3$, $6 = 3 \cdot 2$

$\left(\sqrt[3]{54x}\right)^2 = \left(3\sqrt[3]{2x}\right)^2$

$54 = 9 \cdot 6$, $6 = 2 \cdot 3$

$= 9\sqrt[3]{4x^2}$

f. $(40z)^{\frac{2}{3}} = \left(\sqrt[3]{40z}\right)^2 = \left(2\sqrt[3]{5z}\right)^2$

$40 = 20 \cdot 2$, $20 = 10 \cdot 2$, $10 = 5 \cdot 2$

$4\sqrt[3]{25z^2}$

Steps → ① Rewrite as radical
Power $\frac{a}{b}$ → $\left(\sqrt[b]{\ }\right)^a$

② Factor number. Take out any factors if b of them are under $\sqrt[b]{\ }$.

③ Keep remaining factors under $\sqrt[b]{\ }$.

④ Raise all factors to the a^{th} power.

g. $(250a)^{\frac{2}{3}} = \left(\sqrt[3]{250a}\right)^2 = \left(5\sqrt[3]{2a}\right)^2$

$250 = 25 \cdot 10$, $25 = 5 \cdot 5$, $10 = 5 \cdot 2$

$= 25\sqrt[3]{4a^2}$

h. $(162x)^{\frac{3}{4}} = \left(\sqrt[4]{162x}\right)^3 = \left(3\sqrt[4]{2x}\right)^3$

$162 = 2 \cdot 81$, $81 = 9 \cdot 9$, $9 = 3 \cdot 3$

$2 = 2$

$27\sqrt[4]{8x^3}$

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i. $\sqrt[7]{x^4} = x^{\frac{4}{7}}$ (Rad → Exp; root 7)

j. $\sqrt[5]{x^6} = x^{\frac{6}{5}}$

k. $C = 12\sqrt[3]{n^2} + 1294$

C = cost n = number of chips

What is the cost (C) of producing 251 chips (n)?

$C = 12\sqrt[3]{251^2} + 1294$

$\$6347.43$

$\sqrt[3]{x}$

$\sqrt[4]{x}$

$x^2 \cdot x^4 = x^6$

l. $\left(x^{\frac{7}{12}}\right)\left(x^{\frac{1}{2}}\right) = x^{\frac{7}{12} + \frac{1}{2}\cdot\frac{6}{6} = \frac{7}{12} + \frac{6}{12}}$

$= x^{\frac{13}{12}}$

$= \sqrt[12]{x^{13}} = x\sqrt[12]{x}$

m. $(25x)^{\frac{1}{2}}(64x)^{\frac{1}{2}} = 5 \cdot 8\, x^{\frac{1}{2}} x^{\frac{1}{2}}$

$= 40x$

$\sqrt{25x} \cdot \sqrt{64x}$

$25^{\frac{1}{2}} x^{\frac{1}{2}} 64^{\frac{1}{2}} x^{\frac{1}{2}}$

$5\sqrt{x} \cdot 8\sqrt{x}$

$(5 \cdot 5)^{\frac{1}{2}} x^{\frac{1}{2}} (8 \cdot 8)^{\frac{1}{2}} x^{\frac{1}{2}}$

$5 \cdot 8\sqrt{x \cdot x}$

$5 x^{\frac{1}{2}} \cdot 8 x^{\frac{1}{2}}$

$5 \cdot 8 x$

$5 \cdot 8 x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

$40x$

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n. $(36a)^{\frac{1}{2}}(4a)^{\frac{1}{2}}$

$\sqrt{36a}\ \sqrt{4a}$

$6 \cdot 2\sqrt{a} \cdot \sqrt{a}$

$12a$

o. $\sqrt[4]{x^3} - \sqrt[4]{x^1}$

$x^{\frac{3}{4}} - x^{\frac{1}{4}}$

$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$

$x^2 - x$

p. $\sqrt[3]{2x}\sqrt[3]{2x}\sqrt[3]{2x} = \sqrt[3]{2x \cdot 2x \cdot 2x}$

$= 2x$

$(2x)^{\frac{1}{3}}(2x)^{\frac{1}{3}}(2x)^{\frac{1}{3}}$

$= (2x)^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = (2x)^{\frac{3}{3}} = 2x$