Name _____ Date _____

LESSON 9.1

Study Guide

For use with the lesson "Use Trigonometry with Right Triangles"

GOAL **Use trigonometric functions to find lengths.**

Vocabulary

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The abbreviations *opp*, *adj*, and *hyp* are often used to represent the side lengths of the right triangle. Note that the ratios in the second row are reciprocals of the ratios in the first row:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

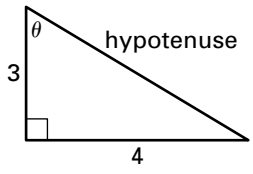
Trigonometric Values for Special Angles

The table below gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

EXAMPLE 1 Evaluate trigonometric functions

Evaluate the six trigonometric functions of the angle θ.



Solution

From the Pythagorean theorem, the length of the hypotenuse is $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$$

Name _____ Date _____

LESSON 9.1

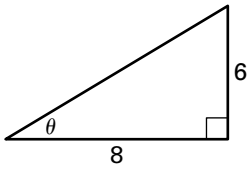
Study Guide *continued*

For use with the lesson "Use Trigonometry with Right Triangles"

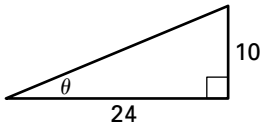
Exercises for Example 1

Evaluate the six trigonometric functions of the angle θ.

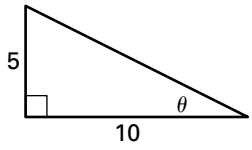
1.



2.

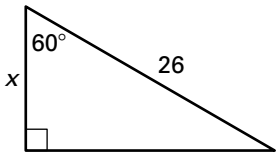


3.



EXAMPLE 2 Find an unknown side length of a right triangle

Find the value of *x* for the right triangle shown.



Solution

Because you are given the side adjacent to $\theta = 60°$ and the hypotenuse, you can write an equation using a trigonometric function that involves the ratio of x and 26. Then solve the equation for x.

$\cos 60° = \frac{\text{adj}}{\text{hyp}}$ Write trigonometric equation.

$\frac{1}{2} = \frac{x}{26}$ Substitute $\frac{1}{2}$ for $\cos 60°$, x for adj, and 26 for hyp.

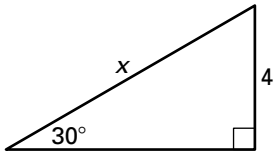
$13 = x$ Multiply each side by 26.

The length of the side is $x = 13$.

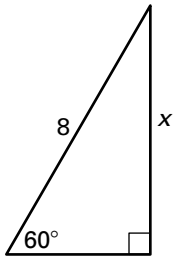
Exercises for Example 2

Find the value of *x* for the right triangle shown.

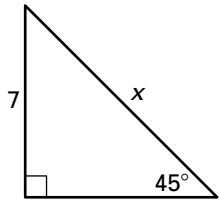
4.



5.



6.



Lesson 9.1 Use Trigonometry with Right Triangles, continued

7. $x \approx 5.87$, $y \approx 9.14$ **8.** $x \approx 1.63$, $y \approx 4.76$

9. $x \approx 11.06$, $y \approx 6.91$ **10.** $B = 51°$, $a \approx 2.59$, $b \approx 3.20$ **11.** $A = 36°$, $b \approx 7.79$, $c \approx 9.62$

12. $B = 63°$, $a \approx 0.51$, $c \approx 1.12$

13. about 6 ft 6 in. **14.** about 48.3 ft

Study Guide

1. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$ **2.** $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\csc\theta = \frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = \frac{12}{5}$

3. $\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = \frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = 2$ **4.** 8

5. $4\sqrt{3}$ **6.** $7\sqrt{2}$

Math and History Application

1. about 12,440 mi **2.** about 21,550 mi

3. 5800 mi shorter **4.** 2200 mi shorter

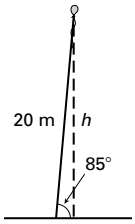
Challenge Practice

1. True, because corresonding sides of similar triangles are proportional. **2.** False. $\tan\theta$ is equal to $\frac{\text{opp}}{\text{adj}}$. You can find the length of the hypotenuse using the Pythagorean theorem, then you can find $\sec\theta$, which is equal to $\frac{\text{hyp}}{\text{adj}}$.

3. False. The angle of elevation and the angle of depression have the same measure because both are formed by a line parallel to the ground and your line of sight and these two angles form alternate interor angles, which by definition are congruent.

4. False. You can also use the cosecant function because $\csc\theta = \frac{\text{hyp}}{\text{opp}}$. **5. a.** *Sample answer:* Because corresponding angles are congruent. **b.** The ratios are equal. **c.** no; no **d.** Yes, because all six trigonometric functions are ratios of the sides of right triangles.

6. a. Let h = the height of the balloon.

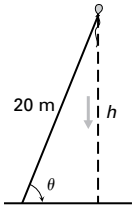


b. $\sin 85° = \frac{h}{20}$; $h \approx 19.9$ m

c. The side of the triangle labeled h will become shorter.

d. 19.7, 18.8, 17.3, 15.3, 12.9, 10.0, 6.8, 3.5

e. As θ approaches 0°, the height h approaches 0.



7. a. $SR = d - x$ **b.** $\frac{\tan R}{\tan P} = \frac{x}{d - x}$

c. $x = \frac{d \tan R}{\tan R + \tan P}$ **d.** $QS = \frac{d \tan P \tan R}{\tan R + \tan P}$

Lesson 9.2 Define General Angles and Use Radian Measure

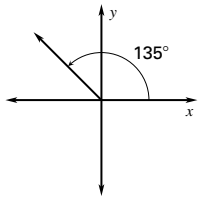
Teaching Guide

1. about 14 m **2.** about 139.6 m^2

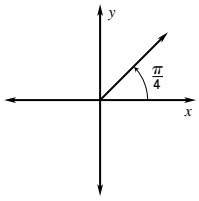
3. about 20.9 m **4.** about 314.2 m^2

Practice Level A

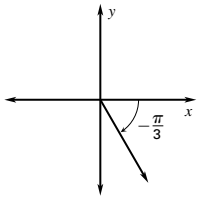
1.



2.



3.



4–7. Sample answers are given. **4.** 450°, −270°

5. 135°, −585° **6.** $\frac{4\pi}{3}$, $-\frac{8\pi}{3}$ **7.** $\frac{13\pi}{4}$, $-\frac{3\pi}{4}$

8. $\frac{2\pi}{3}$ **9.** $-\frac{\pi}{3}$ **10.** 135° **11.** −210°

12. about 3.1 in., about 3.1 $in.^2$ **13.** about 4.2 cm, about 8.4 cm^2 **14.** about 6.3 ft, about 25.1 ft^2