

Practice

$$\textcircled{1} \quad \sqrt{81x^2y^4z^6}$$

$$\textcircled{2} \quad \sqrt{6xy} \cdot \sqrt{10x^2y^5}$$

$$\textcircled{3} \quad 3\sqrt{15a^2} \cdot 5\sqrt{12a}$$

$$\textcircled{4} \quad 16\sqrt[4]{25xy} - 9\sqrt[4]{25xy}$$

$$\textcircled{5} \quad \sqrt[3]{375} - \sqrt[3]{24}$$

$$\textcircled{6} \quad (6 + \sqrt{7})(5 - \sqrt{2})$$

Practice

$$\textcircled{1} \quad \sqrt[3]{81x^2y^4z^6} = \sqrt[3]{\cancel{3 \cdot 3} \cdot \cancel{3 \cdot 3} \cdot x \cdot \cancel{y \cdot y \cdot y} \cdot \cancel{z \cdot z \cdot z}}$$

$\begin{matrix} \uparrow & \uparrow \\ 3 & 3 \end{matrix}$

$3 \cdot 3 \cdot x \cdot y \cdot z \cdot z$
 $9|x|y^2|z^3| \quad 9|x|y^2|z^3|$

$$\textcircled{2} \quad \sqrt{6xy} \cdot \sqrt{10x^2y^5}$$

$$\sqrt{6 \cdot 10 x^3 y^6} = \sqrt{\cancel{2 \cdot 2} \cdot 3 \cdot 5 \cdot \cancel{x \cdot x} \cdot \cancel{y \cdot y \cdot y}}$$

$\begin{matrix} \uparrow & \uparrow \\ 2 & 2 \end{matrix}$

$= 2xyyy\sqrt{3 \cdot 5x}$
 $2|xy^3|\sqrt{15x}$

$$\textcircled{3} \quad 3\sqrt{15a^2} \cdot 5\sqrt{12a}$$

$$3 \cdot 5 \sqrt{15 \cdot 12 a^3} = 15\sqrt{\cancel{2 \cdot 2} \cdot 3 \cdot 3 \cdot 5 \cdot a \cdot a}$$

$\begin{matrix} \uparrow & \uparrow \\ 3 & 3 \end{matrix}$

$15 \cdot 2 \cdot 3 \cdot a \sqrt{5a}$
 $90a\sqrt{5a}$

$$\textcircled{4} \quad 16\sqrt[4]{25xy} - 9\sqrt[4]{25xy}$$

$$(16 - 9)\sqrt[4]{25xy} = 7\sqrt[4]{25xy}$$

$$\textcircled{5} \quad \sqrt[3]{375} - \sqrt[3]{24} = \sqrt[3]{\cancel{3 \cdot 5 \cdot 5 \cdot 5}} - \sqrt[3]{\cancel{2 \cdot 2 \cdot 2 \cdot 3}}$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ 5 & 5 & 5 \\ \uparrow & \uparrow & \uparrow \\ 5 & 5 & 5 \end{matrix}$

$5\sqrt[3]{3} - 2\sqrt[3]{3}$
 $(5-2)\sqrt[3]{3} = 3\sqrt[3]{3}$

$$\textcircled{6} \quad (6 + \sqrt{7})(5 - \sqrt{2})$$

F: $6 \cdot 5 = 30$
 O: $6(-\sqrt{2}) = -6\sqrt{2}$
 I: $(\sqrt{7})(5) = 5\sqrt{7}$
 L: $(\sqrt{7})(-\sqrt{2}) = -\sqrt{14}$
 $-\sqrt{7 \cdot 2}$

$30 - 6\sqrt{2} + 5\sqrt{7} - \sqrt{14}$

Lesson 4.8 Complex Numbers

i^3
 i^4
 i^5
 i^6

imaginary unit

Def: $i = \sqrt{-1}$ the complex number whose square is -1.	F/c: $i^2 = -1$ $\sqrt{-a}$ $(\sqrt{-3})^2$
Ex: $\sqrt{-9} = \sqrt{-1} \cdot \sqrt{9} = i \cdot 3 = 3i$ $\sqrt{-3} = \sqrt{-1} \cdot \sqrt{3} = i\sqrt{3}$	Non-ex. $\sqrt[3]{-8}$ $\sqrt[4]{-1}$

$\sqrt{-24} = 2i\sqrt{6}$ $i^2 = (\sqrt{-1})^2 = (\sqrt{-1} \cdot \sqrt{-1}) = \sqrt{-1 \cdot -1} = -1$

$\sqrt{-18} = 3i\sqrt{2}$

(Handwritten notes for prime factorization of 24 and 18 with arrows pointing to the square root factors.)

Practice

$$\textcircled{1} \quad \sqrt{\frac{25x^4}{36}} = \frac{\sqrt{5 \cdot 5 x x x x}}{\sqrt{6 \cdot 6}} = \frac{5x^2}{6}$$

$$\textcircled{2} \quad \frac{\sqrt{32xy}}{\sqrt{2y}} = \sqrt{\frac{32xy}{2y}} = \sqrt{16x} = 4\sqrt{x}$$

$$\textcircled{3} \quad \frac{3}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} = \frac{3\sqrt{5x}}{\sqrt{5 \cdot 5 x x}} = \frac{3\sqrt{5x}}{5x}$$

$$\textcircled{4} \quad \frac{10}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2 \cdot 2}}{\sqrt[3]{2 \cdot 2}} = \frac{10\sqrt[3]{4}}{2} \rightarrow 5\sqrt[3]{4}$$

$$\textcircled{5} \quad \frac{5}{(4+\sqrt{5})} \cdot \frac{(4-\sqrt{5})}{(4-\sqrt{5})} = \frac{5 \cdot 4 - 5\sqrt{5}}{16-5} = \frac{20-5\sqrt{5}}{11}$$

F : 4 · 4 16

O : 4(-√5) $\begin{array}{|c|} \hline -4\sqrt{5} \\ \hline \end{array}$

I : √5 · 4 $\begin{array}{|c|} \hline 4\sqrt{5} \\ \hline \end{array}$

L : √5(-√5) -5
 $\frac{-\sqrt{5} \cdot 5}{11}$

$$4(4-\sqrt{5}) + \sqrt{5}(4-\sqrt{5})$$

$$16 \quad \begin{array}{|c|} \hline -4\sqrt{5} + 4\sqrt{5} \\ \hline \end{array} \quad -5$$

11

$$(a+b)(a-b) = a^2 - b^2$$

a · a a²

a · -b $\begin{array}{|c|} \hline -ab \\ \hline \end{array}$

b · a $\begin{array}{|c|} \hline +ab \\ \hline \end{array}$

b · -b -b²

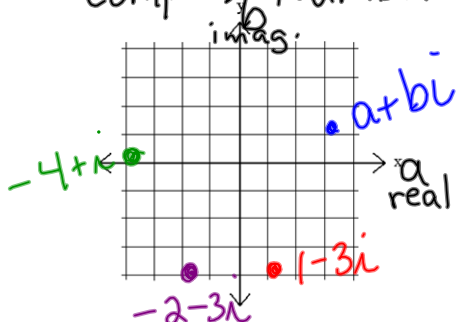
Complex Numbers $(a+bi)$ → standard form

Real Numbers $a+bi$ $b=0$	Imaginary Numbers $a+bi, b \neq 0$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;"> Pure Imaginary Number $0+bi, b \neq 0$ </div>
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* a, b → real numbers

a → "real"
 bi → "imaginary"

Complex Number Plane



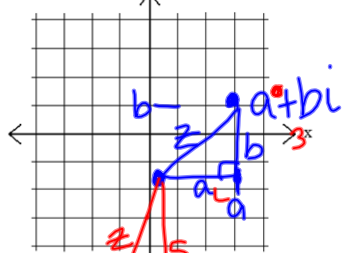
$(a, b) \rightarrow a+bi$

Graph $1-3i \rightarrow (1, -3)$

$-4+i$

$-2-3i \rightarrow (-2, -3)$

absolute value: distance from the origin



$$a^2 + b^2 = z^2$$

$$\sqrt{a^2 + b^2} = \sqrt{z^2}$$

$$\sqrt{a^2 + b^2} = z = |a+bi|$$

$-2-5i$

$$2^2 + 5^2 = z^2$$

$$4 + 25 = z^2$$

$$\sqrt{29} = \sqrt{z^2}$$

$$\sqrt{29} = z$$

$$|4+3i| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Plot $-2-5i$.

Find $|-2-5i| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$

Find $|6| = \sqrt{6^2 + 0^2} = \sqrt{36} = 6$

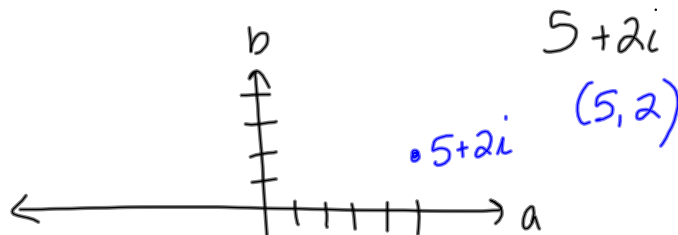
Add	Subtract
Multiply	Divide

$\textcircled{1} (2+i) + (-3+3i)$ $2 + (-3) + i + 3i$ $-1 + 4i$ <hr/> $\textcircled{2} (5+i) + (5-i)$ $5+5+i-i = 10$	$\textcircled{1} (-6-2i) - (4+2i)$ $-6-2i-4-2i$ <p>* Distribute -1 to second ()</p> $-6-4-2i-2i$ $-10-4i$ <hr/> $\textcircled{2} (1+5i) - (3-2i)$
$\textcircled{1} (4i)(2i)$ $-8i^2 = 8(-1) = -8$ $\textcircled{2} -i(4-8i)$ $-4i + 8i^2 = -4i + 8(-1)$ $= -4i - 8 = -8 - 4i$ $\textcircled{3} (5-7i)(-4-3i)$	$\textcircled{1} \frac{1+5i-3+2i}{3} = \frac{-2+7i}{3}$ $\left(\frac{-2+7i}{3}\right) \cdot \frac{(7-2i)}{(7-2i)} = \frac{21-6i}{53}$ $49 - 14i + 14i - 4i^2 = 49 + 4 = 53$ $\textcircled{2} \frac{(5+2i)(3+2i)}{(3-2i)(3+2i)} = \frac{15+10i+6i+4i^2}{9-4i^2} = \frac{11+16i}{13}$

$F \ 5 \cdot (-4) \quad -20$ $O \ 5(-3i) \quad -15i$ $I \ -7i(-4) \quad 28i$ $L \ -7i(-3i) \quad -21i^2$	$F \ 5 \cdot 3 \rightarrow 15$ $O \ 5 \cdot 2i \quad 10i$ $I \ 2i \cdot 3 \quad 6i$ $L \ 2i \cdot 2i \quad 4i^2 = -4$	$\frac{11+16i}{13}$
$\textcircled{4} (3+2i)(-4+13i)$	$F \ 3 \cdot 3 \quad 9$ $O \ 3 \cdot 2i \quad 6i$ $I \ -2i \cdot 3 \quad -6i$ $L \ -2i \cdot 2i \quad -4i^2 = +4$	$\frac{11+16i}{13}$

$$-12 - 5i - 8i - 10i^2 + 10$$

$$-2 - 23i$$



Warm-up - Work with your groups

$$\textcircled{1} \quad \frac{3}{7+2i} \quad \begin{array}{l} \downarrow \quad \downarrow \\ \sqrt{x} \quad \sqrt{x} \\ \sqrt{3} \cdot \sqrt{3} \end{array} = x = \textcircled{3}$$

$$\textcircled{2} \quad x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2} + \frac{1}{2}} = x^{\frac{2}{2}} = x$$

$$\textcircled{3} \quad (x^{\frac{1}{3}})^3 = x^{\frac{1}{3} \cdot 3} = x^1 = x$$

$$x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = x^{\frac{3}{3}} = x$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad \text{or} \quad \left(\sqrt[n]{x} \right)^m$$

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{3}{5}} = \sqrt[5]{x^3}$$

$$m \rightarrow m \quad x^{\frac{1}{m}} = \sqrt[m]{x}$$

11. $\sqrt[3]{250} - \sqrt[3]{54}$

$\begin{matrix} & \wedge & & \\ & 25 & 10 & \\ \textcircled{5} & \textcircled{5} & \textcircled{5} & \textcircled{2} \end{matrix}$

$\begin{matrix} & \wedge & & \\ & 9 & 6 & \\ \textcircled{3} & \textcircled{3} & \textcircled{3} & \textcircled{2} \end{matrix}$

$5\sqrt[3]{2} - 3\sqrt[3]{2}$

$2\sqrt[3]{2}$

4. $\sqrt[3]{\frac{64x^9}{343}}$

$$\frac{\sqrt[3]{64x^9}}{\sqrt[3]{343}}$$

$$\sqrt[3]{\frac{27x^9}{343}}$$

$\sqrt[3]{\frac{3^3 x^9}{7^3}}$
 $\frac{\sqrt[3]{27x^9}}{\sqrt[3]{343}}$
 $\frac{3x^3}{7}$

(3) 9 (3)
 (7) 49
 (27)

