Sec. 9.3 Evaluate Trigonometric Functions of Any Angle

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.

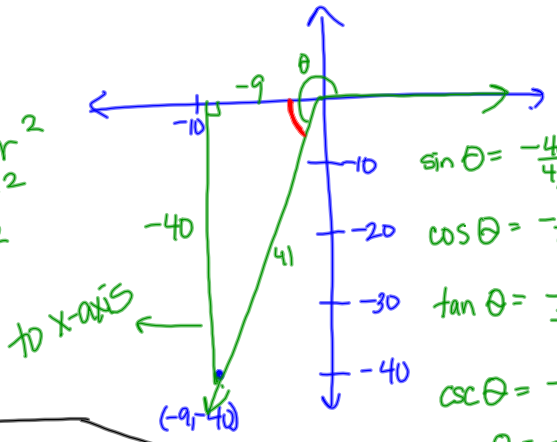
a. $(-9, -40)$

$(-9)^2 + (-40)^2 = r^2$

$81 + 1600 = r^2$

$1681 = r^2$

$41 = r$



$\sin\theta = \frac{-40}{41}$

$\cos\theta = \frac{-9}{41}$

$\tan\theta = \frac{-40}{-9} = \frac{40}{9}$

$\csc\theta = -\frac{41}{40}$

$\sec\theta = -\frac{41}{9}$

$\cot\theta = \frac{9}{40}$

Point (x, y) Radius r

Use $x^2 + y^2 = r^2$ to find r

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

* $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} \cdot \frac{hyp}{adj} = \frac{opp}{adj}$

$\frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$

b. $(\overset{x}{3}, \overset{y}{-3})$

$(3)^2 + (-3)^2 = r^2$

$9 + 9 = r^2$

$\sqrt{18} = \sqrt{r^2}$

$3\sqrt{2} = r$

$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$

$\csc\theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\sec\theta = \sqrt{2}$

$\cot\theta = -1$

Feb 12-8:31 AM

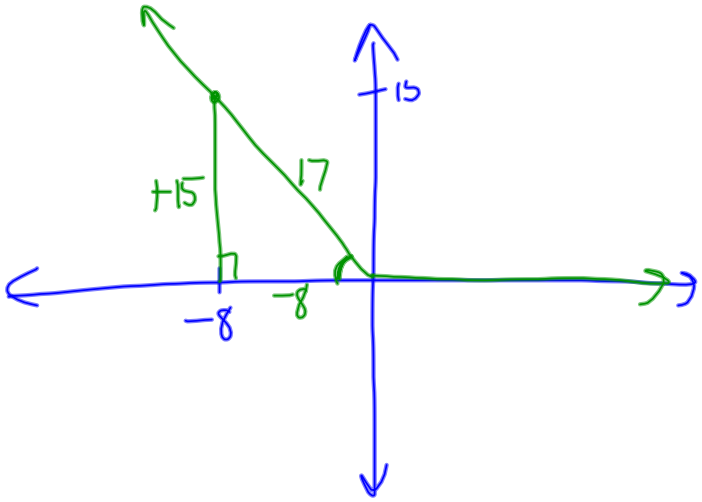
c. $(-8, 15)$ (x, y)

$(-8)^2 + (15)^2 = r^2$

$64 + 225 = r^2$

$289 = r^2$

$17 = r$



$\sin\theta = \frac{y}{r} = \frac{15}{17}$ $\quad \csc\theta = \frac{17}{15}$

$\cos\theta = \frac{x}{r} = -\frac{8}{17}$ $\quad \sec\theta = -\frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\quad \cot\theta = -\frac{8}{15}$

Evaluate the six trigonometric functions of θ.

a. $\theta = 210^\circ$

$\sin 210^\circ = -\frac{1}{2}$

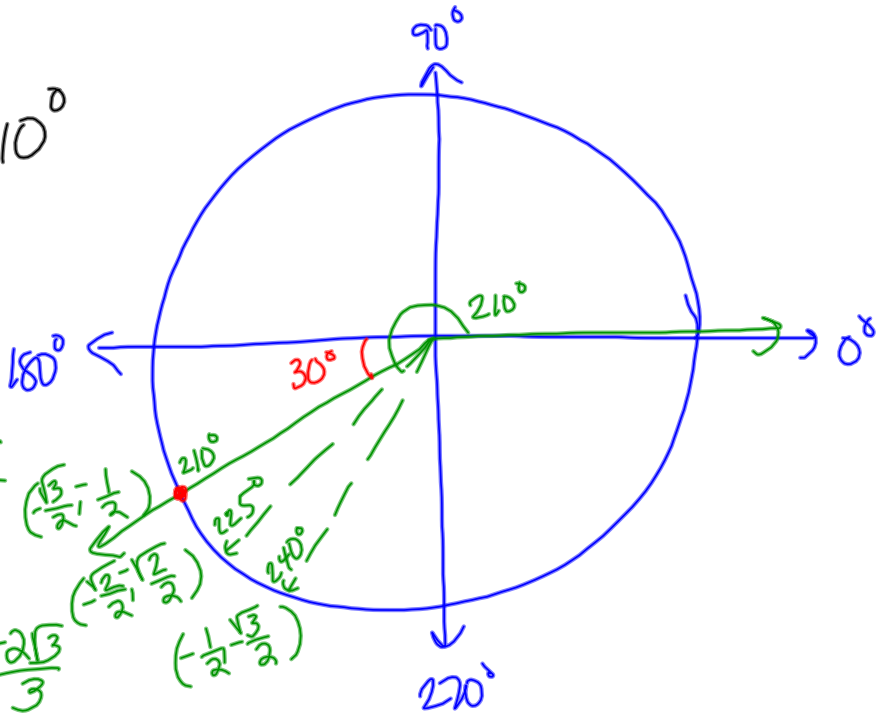
$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

$\tan 210^\circ = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc 210^\circ = -2$

$\sec 210^\circ = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\cot 210^\circ = \sqrt{3}$



b. $\theta = 2\pi$

$\sin 2\pi = 0$

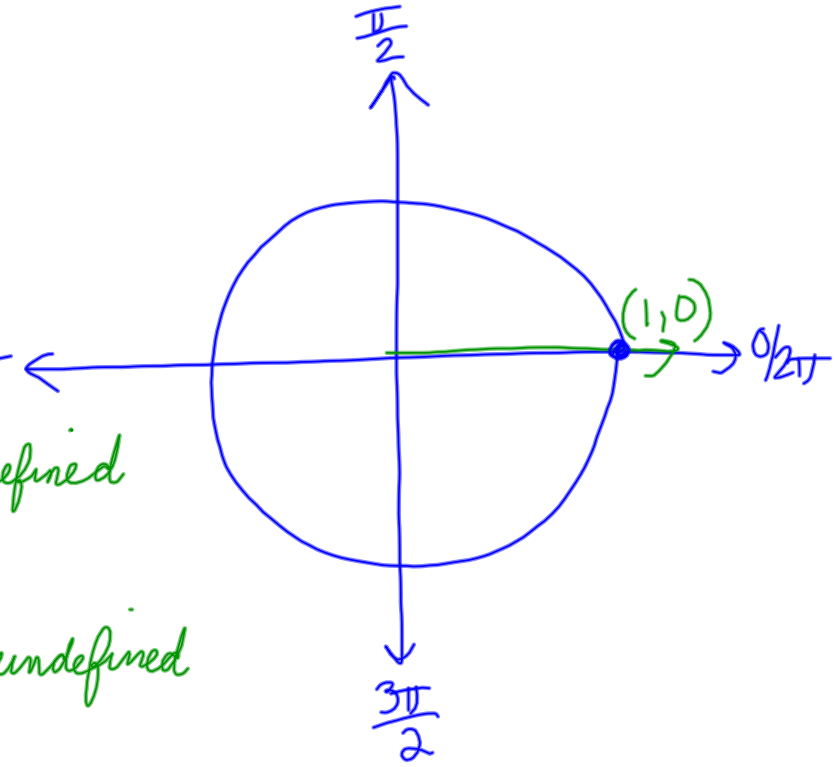
$\cos 2\pi = 1$

$\tan 2\pi = \frac{0}{1} = 0$

$\csc 2\pi = \frac{1}{0} \rightarrow$ undefined

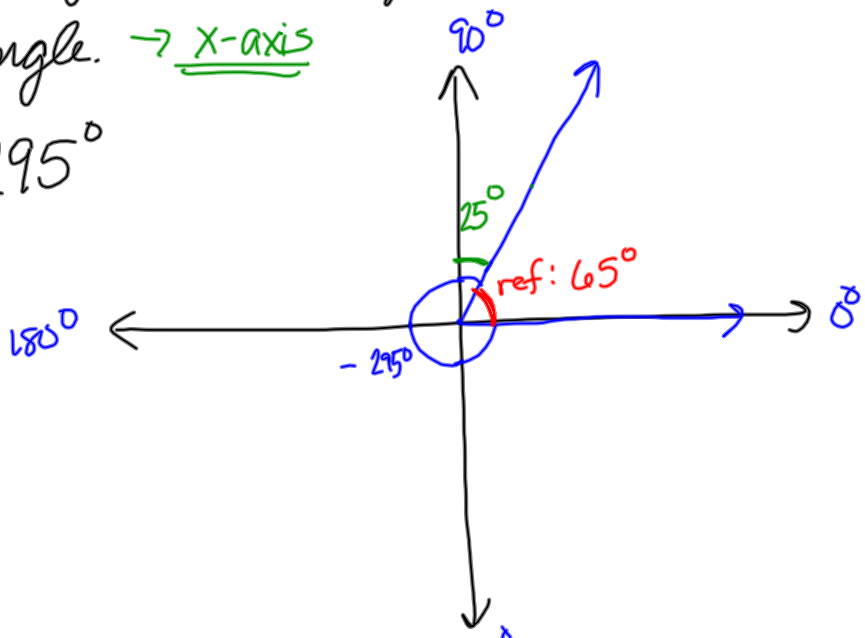
$\sec 2\pi = 1$

$\cot 2\pi = \frac{1}{0} \rightarrow$ undefined

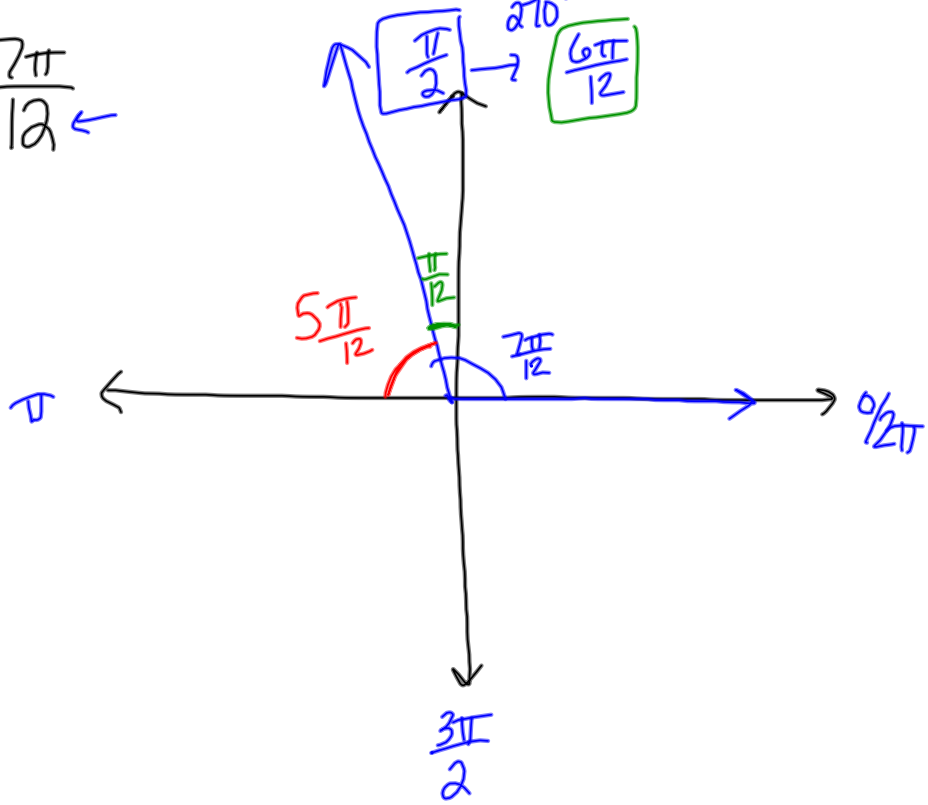


Sketch the angle. Then find its reference angle. → x-axis

a. $\theta = -295^\circ$



b. $\theta = \frac{7\pi}{12}$ ←



Feb 12-9:47 AM