Sec. 4.6 Solve Exponential and Logarithmic Equations

Recall:

If $y = b^x$, then $\log_b y = x$ (exponent)

$b^{\log_b x} = x$ $\quad$ $\log_b b^x = x$

$3^{\log_3 9} = 9$ $\quad$ $\log_3 3^2 = 2$

3^2 → 9

$\log_5 125 = 3$

$5^3 = 125$

$5 \cdot 25$

$5 \cdot 5 \cdot 5$

Properties:

$\log_b m + \log_b n = \log_b m \cdot n$

$\log_b m - \log_b n = \log_b \frac{m}{n}$

$n \log_b m = \log_b m^n$

* Change of Base

$$\log_a c = \frac{\log_b c}{\log_b a} = \frac{\log c}{\log a} = \frac{\ln c}{\ln a}$$

Apr 23-10:08 AM

Solve the equation.

1. $3^x = \left(\frac{1}{9}\right)^{x+3}$ → Bases need to be equal

$3^x = \left(\frac{1}{3^2}\right)^{x+3}$

$3^x = \left(3^{-2}\right)^{(x+3)}$

$3^x = 3^{-2x-6}$

$\log_3 3^x = \log_3 3^{(-2x-6)}$

$x = -2x - 6$

$+2x \quad +2x$

$\frac{3x}{3} = \frac{-6}{3}$

$x = -2$

b. $9^x = 35 \rightarrow$ log form $\log_9 35 = x$

$\log_9 9^x = \log_9 35$

$x = \log_9 35$ or $\frac{\log 35}{\log 9}$

≈ 1.62

c. ① $\log_4(2x+8) = \log_4(6x-12)$

② $2x + 8 = 6x - 12$

$-2x \quad -2x$

$8 = 4x - 12$

$+12 \quad +12$

$\frac{20}{4} = \frac{4x}{4}$

$5 = x$

Check for E.S.

d. $\log_7(3x-2) = 2$

$3x-2 = 49$

$+2 \quad +2$

$\frac{3x}{3} = \frac{51}{3}$

$x = 17$

* Check

e. $\log_6 3x + \log_6(x-4) = 2$

Condense into one expression

$\log_b m + \log_b n = \log_b m \cdot n$

$\log_6 3x(x-4) = 2$

$6^2 = 3x(x-4)$

$3x(x-4) = 36$

$\frac{3x^2}{3} - \frac{12x}{3} = \frac{36}{3}$

$x^2 - 4x = 12$

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

$x = -2, 6$

Apr 23-10:32 AM