

Sec. 3.6 Solve Radical Equations

Solve

a. $\sqrt{5x-9} = 11$

$$(\sqrt{5x-9})^2 = 11^2$$

$$5x - 9 = 121$$

$$\begin{array}{r} +9 \\ +9 \end{array}$$

$$\hline 5x = 130$$

$$\frac{5x}{5} = \frac{130}{5}$$

$$x = 26$$

b. $\frac{7x^{\frac{3}{5}}}{7} = \frac{56}{7}$

$$x^{\frac{3}{5} \cdot \frac{5}{3}} = 8^{1 \cdot \frac{5}{3}} \rightarrow$$

$$\sqrt[5]{x^3} = 8$$

$$x^3 = 8^5$$

$$\sqrt[3]{x^3} = \sqrt[3]{8^5}$$

$$x = 2^5$$

$$x = 32$$

$$\left[x^{\frac{3}{5}}\right]^{\frac{5}{3}} = 8^{\frac{5}{3}}$$

$$x^1 = \left(\sqrt[3]{8}\right)^5$$

$$x = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$x = 32$$

$$c. (x+3)^{\frac{5}{2}} = 32 \rightarrow \sqrt{(x+3)^5} = 32$$

$$\left[(x+3)^{\frac{5}{2}} \right]^{\frac{2}{5}} = 32^{\frac{2}{5}} \quad (\sqrt{(x+3)^5} = 32^2)$$

$$(x+3)^{\textcircled{5}} = 32^2$$

$$x+3 = \left(\sqrt[5]{32} \right)^2 \quad \sqrt[5]{(x+3)^5} = \sqrt[5]{32^2}$$

$$x+3 = 2^2$$

$$x+3 = 2^2$$

$$x+3 = 4$$

$$x+3 = 4$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$x = 1$$

$$x = 1$$

$$d. \sqrt{x-6} = x-8$$

$$\left(\sqrt{x-6} \right)^2 = (x-8)^2 \rightarrow \begin{array}{l} (x-8)(x-8) \\ x^2 - 8x - 8x + 64 \end{array}$$

$$\begin{array}{r} x-6 = x^2 - 16x + 64 \\ -x+6 \quad \quad \quad -x+6 \\ \hline \end{array}$$

$$0 = x^2 - 17x + 64$$

$$0 = (x-7)(x-10)$$

$$\begin{array}{l} 1 \cdot 70 \\ 2 \cdot 35 \\ 5 \cdot 14 \\ \boxed{7 \cdot 10} \end{array}$$

$$x-7=0$$

$$x-10=0$$

$$\begin{array}{r} +7 \quad +7 \\ \hline \end{array}$$

$$\begin{array}{r} +10 \quad +10 \\ \hline \end{array}$$

$$x = 7$$

$$x = 10$$

Check for extraneous solutions

$$\sqrt{x-6} = x-8$$

$$\sqrt{7-6} = 7-8$$

$$\sqrt{10-6} = 10-8$$

$$\sqrt{1} = -1$$

$$\sqrt{4} = 2$$

$$1 = -1$$

$$2 = 2$$

$$d. \sqrt{x+6} = \sqrt{11-x} - 3$$

$$(\sqrt{x+6})^2 = (\sqrt{11-x} - 3)^2$$

$$x+6 = \frac{(\sqrt{11-x}-3)(\sqrt{11-x}-3)}{\sqrt{11-x} \cdot \sqrt{11-x}} = \frac{11-x - 3\sqrt{11-x} - 3\sqrt{11-x} + 9}{11-x}$$

$$x+6 = 11-x - 6\sqrt{11-x} + 9$$

$$\frac{2x-14}{2} = \frac{-6\sqrt{11-x}}{2}$$

$$(x-7)(x-7) \quad x-7 = -3\sqrt{11-x}$$

$$x^2-14x+49 = (-3\sqrt{11-x})^2$$

$$x^2 - 14x + 49 = 9(11-x)$$

$$x^2 - 14x + 49 = 99 - 9x$$

$$+9x - 99 \quad -99 + 9x$$

$$x^2 - 5x - 50 = 0 \quad \boxed{5 \cdot 10}$$

$$(x+5)(x-10) = 0$$

$$x+5=0 \quad x-10=0$$

$$\frac{-5 \quad -5}{\quad} \quad \frac{+10 \quad +10}{\quad}$$

$$\boxed{x=-5} \quad \cancel{x=10}$$

E.S.:

$$\sqrt{x+6} = \sqrt{11-x} - 3$$

$$\sqrt{-5+6} = \sqrt{11-(-5)} - 3$$

$$\sqrt{1} = \sqrt{16} - 3$$

$$1 = 4 - 3$$

$$1 = 1$$

$$\sqrt{10+6} = \sqrt{11-10} - 3$$

$$\sqrt{16} = \sqrt{1} - 3$$

$$4 = 1 - 3$$

$$4 \neq -2$$

$$15. f(x) = x^4 + x^3 + 2x^2 + 4x - 8$$

$$\text{p.r. } z : \pm 1, \pm 2, \pm 4, \pm 8$$

$$\begin{array}{r|rrrrrr} 1 & 1 & 1 & 2 & 4 & -8 & \\ & & 1 & 2 & 4 & 8 & \\ \hline & 1 & 2 & 4 & 8 & 0 & \\ & & 1 & 3 & 7 & & \\ \hline & 1 & 3 & 7 & & & \end{array} \quad \boxed{x=1}$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & 4 & 8 \\ & & 2 & 8 & \\ \hline & 1 & 4 & & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 4 & 8 \\ & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 0 \end{array} \quad \boxed{x=-2}$$

$$x^2 + 0x + 4 = 0$$

$$\begin{array}{r} x^2 + 4 = 0 \\ -4 \quad -4 \\ \hline \end{array}$$

$$x^2 = -4$$

$$\sqrt{x^2} = \pm \sqrt{-4}$$

$$x = \pm 2i$$

$$x = 1, -2, 2i, -2i$$

$$x = 1$$

$$(x-1) > 0$$

$$* x^2 + 4 = 0$$

$$(x - 2i)(x + 2i) = 0$$

$$x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$