Sec. 10.3 Verify Trigonometric Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\frac{O}{H} \cdot \frac{H}{A}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

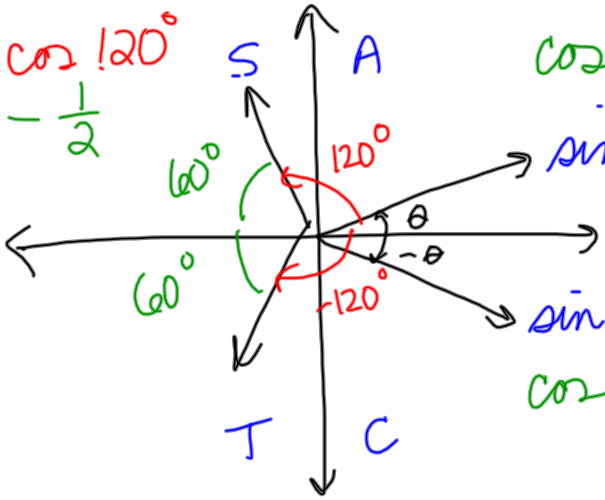
Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\tan(-\theta) = -\tan\theta$

$\cos(-\theta) = \cos\theta$

$\cos(-120^\circ) = \cos 120^\circ$

$-\frac{1}{2} = -\frac{1}{2}$



$\cos 30^\circ = \frac{\sqrt{3}}{2}$

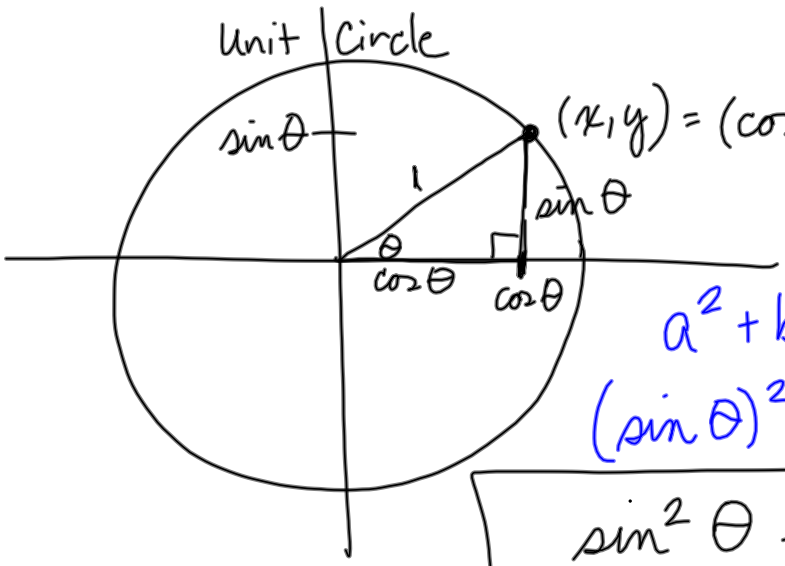
$\sin 30^\circ = \frac{1}{2}$

$\sin(-30^\circ) = -\frac{1}{2}$

$\cos(-30^\circ) = \frac{\sqrt{3}}{2}$

60	$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
45	$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
30	$(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Pythagorean Identities



$a^2 + b^2 = c^2$

$(\sin\theta)^2 + (\cos\theta)^2 = 1^2$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Use a Pythagorean identity to find $\cos(\theta)$ if θ is in quadrant IV.

$$\tan\theta = -\frac{\sqrt{7}}{4}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \left(-\frac{\sqrt{7}}{4}\right)^2 = \sec^2\theta$$

$$1 + \frac{7}{16} = \sec^2\theta$$

$$\frac{16}{16} + \frac{7}{16} = \sec^2\theta$$

$$\frac{23}{16} = \sec^2\theta$$

$$\frac{\sqrt{23}}{4} = \sec\theta$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\cos\theta = \frac{4}{\sqrt{23}} = \boxed{\frac{4\sqrt{23}}{23}}$$

S	A
T	C