

## Sec. 4.5 Properties of logarithms

Let  $b$ ,  $m$ , and  $n$  be positive numbers such that  $b \neq 1$ .

Product Property:  $\log_b mn = \log_b m + \log_b n$

$$\log_2 8 \cdot 4 = \log_2 8 + \log_2 4$$

$$\log_2 32 = 3 + 2$$

$$\boxed{5 = 5}$$

Quotient Property:  $\log_b \frac{m}{n} = \log_b m - \log_b n$

$$\log_3 \frac{81}{9} = \log_3 81 - \log_3 9$$

$$\log_3 9 = 4 - 2$$

$$2 = 2$$

Power Property:  $\log_b m^n = n \cdot \log_b m$

$$\log_5 25^3 = 3 \log_5 25$$

$$\log_5 (5^2)^3 = 3 \cdot 2$$

$$\log_5 5^6 = 6$$

$$6 = 6$$

Use properties of logarithms.

Use  $\log_3 5 \approx 1.465$  and  $\log_3 6 \approx 1.631$   
to evaluate the logarithm.

$$\begin{aligned} \text{a. } \log_3 \frac{6}{5} &= \log_3 6 - \log_3 5 \\ &\approx 1.631 - 1.465 \\ &= 0.166 \end{aligned}$$

$$\begin{aligned} \text{b. } \log_3 30 &= \log_3 5 + \log_3 6 \\ &\approx 1.465 + 1.631 \\ &= 3.096 \end{aligned}$$

$$\begin{aligned} \text{c. } \log_3 36 &= \log_3 6^2 \\ &= 2 \log_3 6 \\ &\approx 2(1.631) \\ &= 3.262 \end{aligned}$$

Use  $\log_2 7 \approx 2.807$  and  $\log_2 3 \approx 1.585$   
to evaluate

$$\text{a. } \log_2 \frac{7}{3}$$

$$\text{b. } \log_2 21$$

$$\text{c. } \log_2 49$$

$$\text{d. } \log_2 27$$

Expand a logarithmic expression

Expand  $\log_8 \frac{(3x^4)y}{y}$

$$\begin{aligned} &\log_8 3x^4 - \log_8 y \\ &\log_8 3 + \log_8 x^4 - \log_8 y \\ &\log_8 3 + 4 \log_8 x - \log_8 y \quad \text{Power} \end{aligned}$$

Condense a logarithmic expression

$$\begin{aligned} \text{Condense } \ln x + 3 \ln y - 2 \ln 4 & \\ \ln x + \ln y^3 - \ln 4^2 & \quad \text{Power} \\ \ln xy^3 - \ln 16 & \\ \ln \frac{xy^3}{16} & \end{aligned}$$

Practice

$$\text{a. Expand } \ln x^5 y^2 z$$

$$\text{b. Condense } 8 \log x + \frac{1}{2} \log y$$