

Sec. 4.4 Evaluate Logarithms

Let $b > 0$, $y > 0$, $b \neq 1$

The logarithm of y with base b is

$\log_b y = x$ if and only if

$b^x = y$

LOG says
"What is the exponent of b that produces y ?"

Rewrite logarithmic equations

Logarithmic form
 $\log_b y = x$

Exponential form
 $b^x = y$

a. $\log_2 16 = 4$

$2^4 = 16$

b. $\log_4 1 = 0$

$4^0 = 1$

c. $\log_9 9^1 = 1$

$9^1 = 9$

d. $\log_{\frac{1}{5}} 25 = -2$

$(\frac{1}{5})^{-2} = 25$

Practice:

Rewrite in exponential form:

a. $\log_5 125 = 3$

$5^3 = 125$

b. $\log_{11} 11 = 1$

$11^1 = 11$

c. $\log_8 1 = 0$

$8^0 = 1$

d. $\log_{\frac{1}{3}} 27 = -3$

$(\frac{1}{3})^{-3} = 27$

Evaluate logarithms

$$a. \log_3 81 = 4$$

$\begin{array}{c} \wedge \\ 9 \quad 9 \\ \textcircled{3} \textcircled{3} \textcircled{3} \end{array}$

$$\log_3 3^4 = 4$$

$$b. \log_4 0.25 = \log_4 \frac{1}{4} = \log_4 4^{-1} = -1$$

$$c. \log_{\frac{1}{6}} 36 = \log_{\frac{1}{6}} 6^2 = \log_{\frac{1}{6}} \left(\frac{1}{6}\right)^{-2} = -2$$

$$\log_{\frac{1}{6}} 36 = -2$$

$$d. \log_{27} 3$$

$$\log_{\boxed{27}} \underline{3} = \frac{1}{3}$$

Practice:

$$a. \log_6 216 = 3$$

$$b. \log_5 \frac{1}{25} = -2 \quad \log_5 \frac{1}{5^2} = \log_5 5^{-2} = -2$$

$$c. \log_{\frac{1}{2}} 64 = -6 \quad \log_{\frac{1}{2}} 2^6 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-6} = -6$$

$$d. \log_{32} 2 = \frac{1}{5}$$

$$\log_{32} 2 = \log_{32} \sqrt[5]{32} = \log_{32} 32^{\frac{1}{5}} = \frac{1}{5}$$

Use inverse properties $\log_3 9 = P = 2$
 $3^P = 9$

$$\log_a a^x = x$$

$$a^{\log_a x} = x$$

$$\log_3 3^2 = 2$$

$$\text{ex: } 3^{\log_3 9} = 9$$

$$\log_3 9 = 2$$

$$3^2 = 9$$

* Note $\log \rightarrow \log_{10}$ common logarithm
 $\ln \rightarrow \log_e$ natural logarithm

Examples:

Simplify

$$a. 10^{\log_{10} 9.6} = 9.6$$

$$b. \log_7 49^{2x} = \log_7 (7^2)^{2x}$$

$$= \log_7 7^{4x} = 4x$$

$$c. e^{\ln 4.5} = e^{\log_e 4.5} = 4.5$$

$$d. \ln e^{-2x} = -2x$$

$$e. \log_4 4^{7x} = 7x$$

$$f. 10^{\log 3x} = 3x$$

Sec. 4.5 Properties of logarithms

Let b , m , and n be positive numbers such that $b \neq 1$.

Product Property: $\log_b mn = \log_b m + \log_b n$

$$\log_2 8 \cdot 4 = \log_2 8 + \log_2 4$$

$$\log_2 \underbrace{32}_{2^5} = 5 \leftarrow \begin{matrix} \downarrow \\ 3 + 2 \end{matrix}$$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^n = n \cdot \log_b m$

$$15. \log_{\underline{16}} \frac{1}{4} = \log_{16} 4^{-1} = \log_{16} \sqrt[16]{16}^{-1}$$

$$\log_{16} 16^{-\frac{1}{2}} = -\frac{1}{2}$$

$$16. \log_{\frac{1}{4}} 16 \leftarrow$$

$$\log_{\frac{1}{4}} 4^2 = \log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{-2} = -2$$

57 \$ 2000 4% CC
 P .04=r
 t=5 yrs

$$A = P e^{rt}$$

$$A = 2000 e^{0.04(5)}$$

$$A = \$ 2442.81$$

58. \$ 1114.17

31.

\$ 200

dec. 25% = r

$$y = a(1-r)^t$$

$$y = 200(1-.25)^t$$

$$y = 200(.75)^t$$

$$\frac{100}{200} = \frac{200(.75)^t}{200}$$

$$y = 200(.75)^3$$

$$\frac{1}{2} = .75^t$$

$$\approx \$84.38$$