Chapter 4

Exponential Growth Function

$f(x) = a \cdot b^x, \quad b > 1$

Ex: $y = 2^x$

$f(1) = 2^1 = 2$

$f(10) = 2^{10} = 1024$

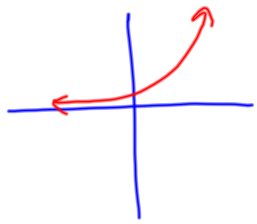
$y = 2^{-1} = \frac{1}{2^1} = \frac{1}{2} = 0.5$

$y = 2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024} \approx 0.00098$

Exponential Decay Model

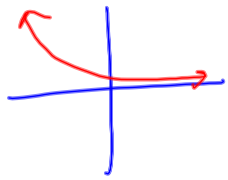
$f(x) = a \cdot b^x, \quad 0 < b < 1$

Ex: $f(x) = \left(\frac{1}{2}\right)^x$



$f(1) = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$f(10) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$



$(-5)^0 = 1$

$(-5)^1 = -5$

$(-5)^2 = 25$

$(-5)^3 = -125$

$* \; b > 0$

Exponential Growth Models

$y = a(1+r)^t$,

$a \rightarrow$ initial amount

$r \rightarrow$ percent increase (decimal)

$t \rightarrow$ number of years

* Decay

$y = a(1-r)^t$

Ex: In 1970, 330,000 people lived in Kern County, CA. From 1970 to 2000, the population grew at an average annual rate of about 2.4%.

1 – Write the exponential growth model.

2 – Estimate the population of Kern County in 1990.

$y = a(1+r)^t$

$y = 330{,}000(1+0.024)^t$

1 $y = 330{,}000(1.024)^t$

$t = 20$ $y = 330{,}000(1.024)^{20}$

$y \approx 530{,}290$

about 530,000 people

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Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$P \rightarrow$ initial principal

$r \rightarrow$ annual rate (decimal)

$n \rightarrow$ number of times compounded each year

$t \rightarrow$ number of years

You deposit \$5500 in an account that pays 3.6% annual interest. Find the balance after 2 years if interest is compounded:

a. semiannually

b. monthly

a. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$\boxed{\begin{aligned} P &= 5500 \\ r &= 0.036 \\ t &= 2 \end{aligned}}$$

$n = 2$

$A = 5500\left(1 + \frac{0.036}{2}\right)^{2 \cdot 2}$

$A = 5500(1.018)^4$

\$5906.82

b. $A = 5500\left(1 + \frac{0.036}{12}\right)^{12 \cdot 2}$ $n = 12$

$A = 5500(1.003)^{24}$

$A =$ \$5909.97

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Continuously Compounded Interest

The Natural Base e

As n approaches $+\infty$,

$\left(1 + \frac{1}{n}\right)^n$ approaches

$e \approx 2.718281828.$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $n \to \infty$

$$\boxed{A = Pe^{rt}}$$

Ex: deposit \$3,000 in 3.5% annual interest account compounded continuously. What is the balance after 3 years?

e^x

$\boxed{\ln}$

$A = Pe^{rt}$

$A = 3000e^{0.035(3)}$

$3000e^{0.105}$

\$3332.13

Ex:
#55
p.249

$y = 1.28e^{1.31x}$

$x \to$ # yrs since 1997

2002?

$x = 5$

$y = 1.28e^{1.31(5)}$

$y = 1.28e^{6.55}$

895 million camera phones

Jan 6-8:59 AM

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