Sec. 9.3 Evaluate Trigonometric Functions of Any Angle

Use the given point on the terminal side of an angle θ in standard position to evaluate the six trigonometric functions of θ.

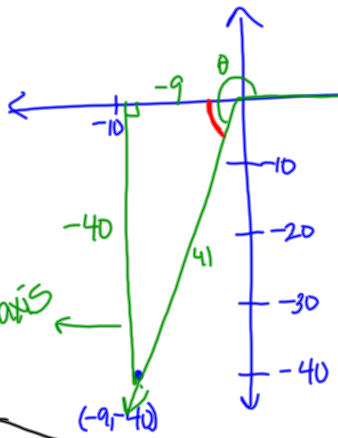
a. $(-9, -40)$

$(-9)^2 + (-40)^2 = r^2$

$81 + 1600 = r^2$

$1681 = r^2$

$41 = r$



$\sin\theta = -\frac{40}{41}$

$\cos\theta = -\frac{9}{41}$

$\tan\theta = \frac{-40}{-9} = \frac{40}{9}$

$\csc\theta = -\frac{41}{40}$

$\sec\theta = -\frac{41}{9}$

$\cot\theta = \frac{9}{40}$

Point (x, y)	Radius r

Use $x^2 + y^2 = r^2$ to find r

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

* $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{\cancel{hyp}}}{\frac{adj}{\cancel{hyp}}} \cdot \frac{\frac{hyp}{adj}}{\frac{hyp}{adj}} = \frac{opp}{adj}$

$\frac{\frac{y}{\cancel{r}}}{\frac{x}{\cancel{r}}} = \frac{y}{x}$

b. $(\overset{x}{3}, \overset{y}{-3})$

$(3)^2 + (-3)^2 = r^2$

$9 + 9 = r^2$

$\sqrt{18} = \sqrt{r^2}$

$3\sqrt{2} = r$

$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$

$\csc\theta = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\sec\theta = \sqrt{2}$

$\cot\theta = -1$

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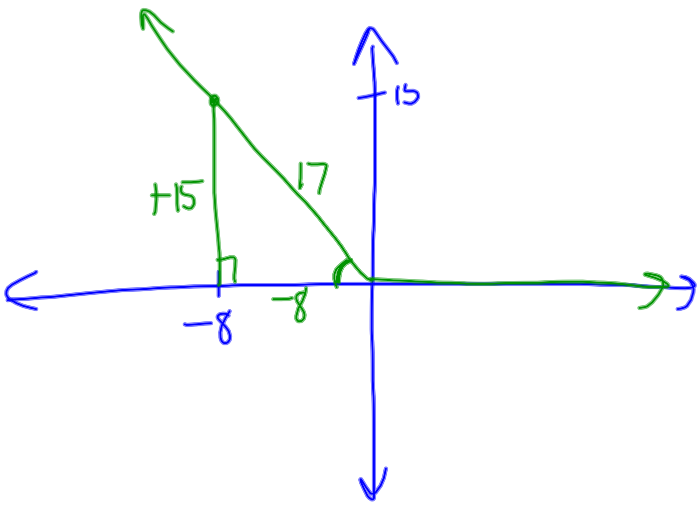
c. $(\overset{x}{-8}, \overset{y}{15})$

$(-8)^2+(15)^2=r^2$

$64+225=r^2$

$289=r^2$

$17=r$



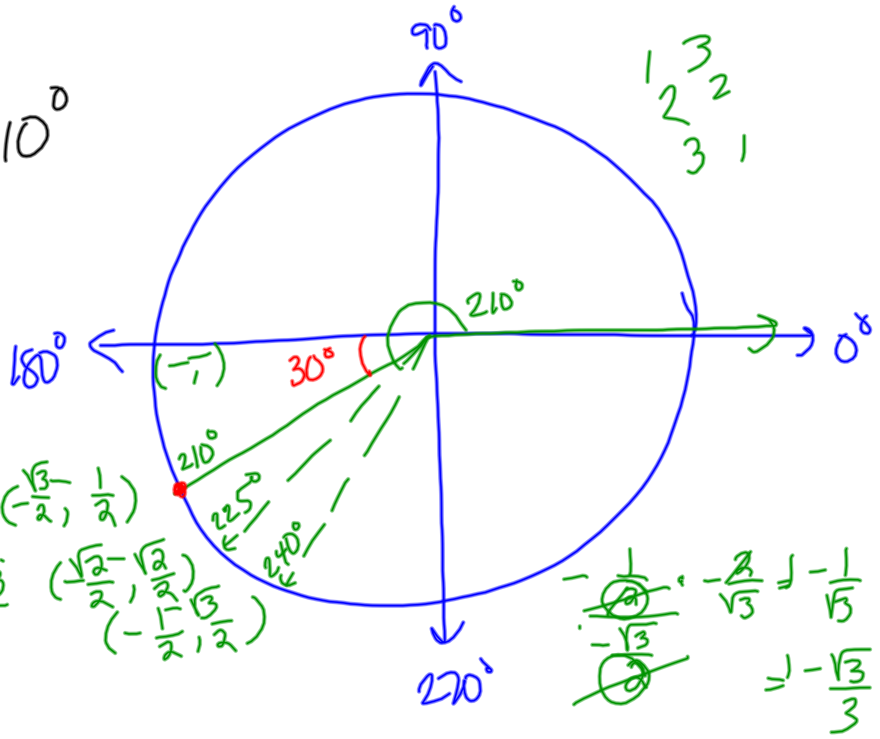
$\sin\theta=\frac{y}{r}=\frac{15}{17}$ $\quad\csc\theta=\frac{17}{15}$

$\cos\theta=\frac{x}{r}=-\frac{8}{17}$ $\quad\sec\theta=-\frac{17}{8}$

$\tan\theta=\frac{y}{x}=-\frac{15}{8}$ $\quad\cot\theta=-\frac{8}{15}$

Evaluate the six trigonometric functions of θ.

a. $\theta = 210^\circ$



$\sin 210^\circ = -\frac{1}{2}$

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

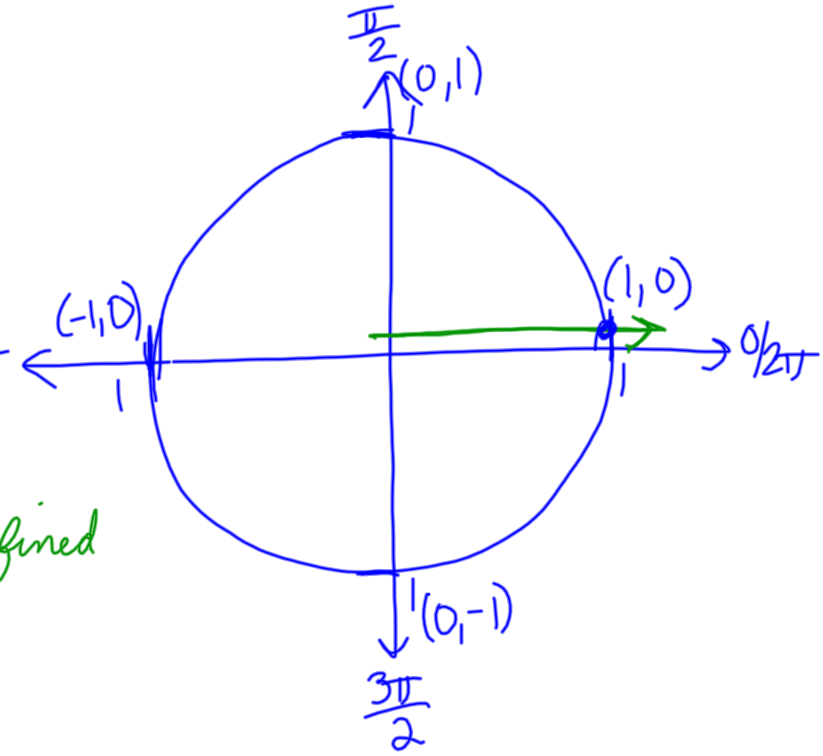
$\tan 210^\circ = \frac{-1}{-\sqrt{3}} = +\frac{\sqrt{3}}{3}$

$\csc 210^\circ = -2$

$\sec 210^\circ = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\cot 210^\circ = \sqrt{3}$

b. $\theta = 2\pi$



$\sin 2\pi = 0$ $\quad \frac{0}{1}$

$\cos 2\pi = 1$

$\tan 2\pi = \frac{0}{1} = 0$

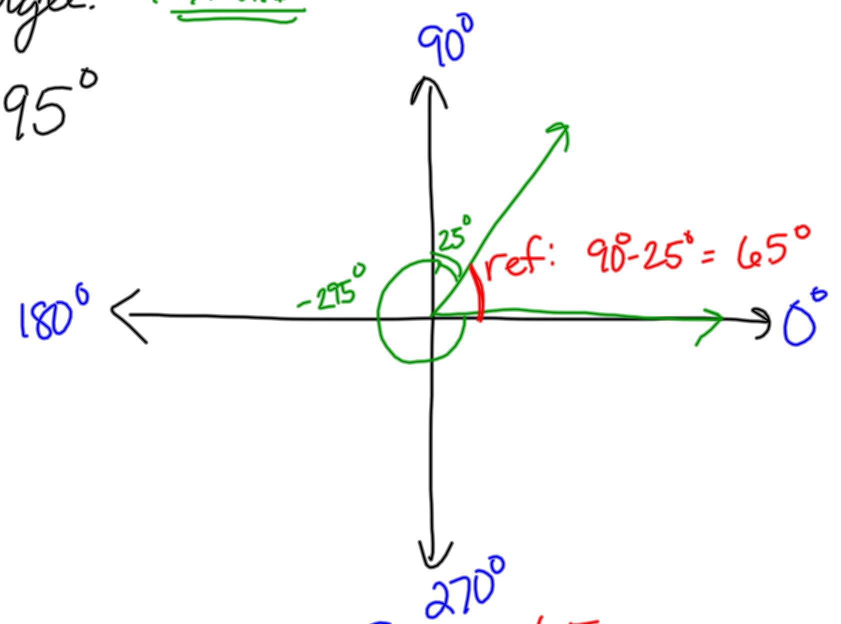
$\csc 2\pi = \frac{1}{0} =$ undefined

$\sec 2\pi = 1$

$\cot 2\pi = \frac{1}{0} \rightarrow$ undefined

Sketch the angle. Then find its reference angle. → x-axis

a. $\theta = -295^\circ$

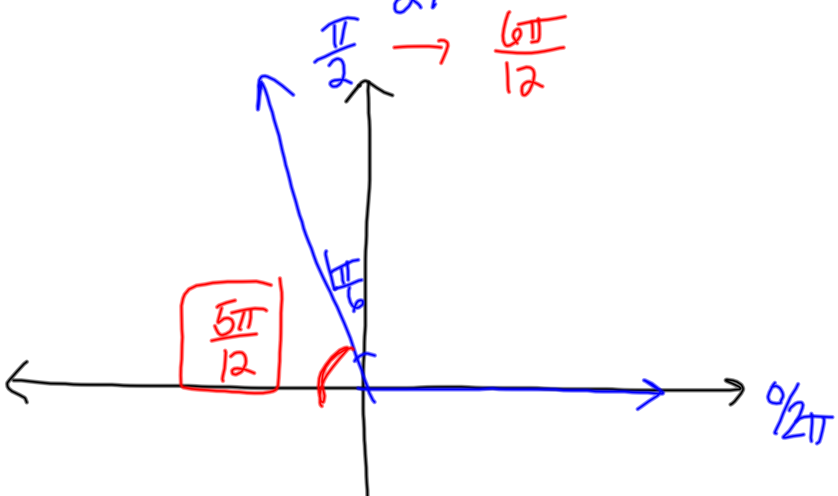


b. $\theta = \frac{7\pi}{12}$

$\pi - \frac{7\pi}{12}$

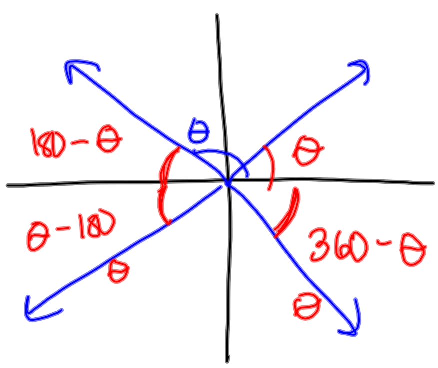
$\frac{12\pi}{12} - \frac{7\pi}{12} = \frac{5\pi}{12}$

$\frac{12\pi}{12}$ π



$\frac{3\pi}{2}$

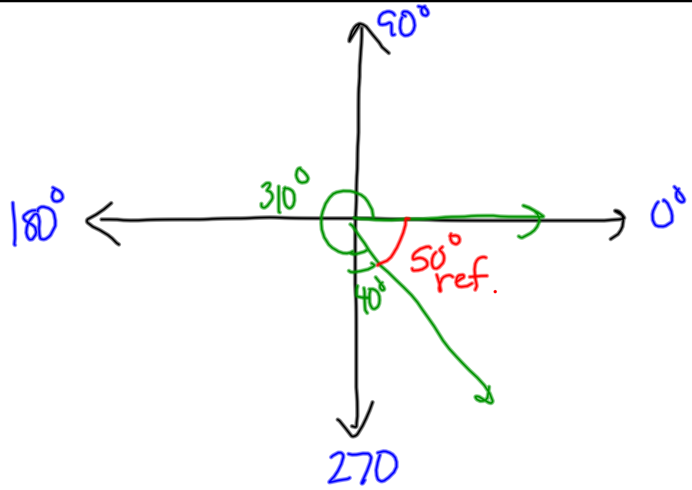
$0^\circ \le \theta \le 360^\circ$	$0 \le \theta \le 2\pi$
QI → θ	θ
QII → $180 - \theta$	$\pi - \theta$
QIII → $\theta - 180$	$\theta - \pi$
QIV → $360 - \theta$	$2\pi - \theta$



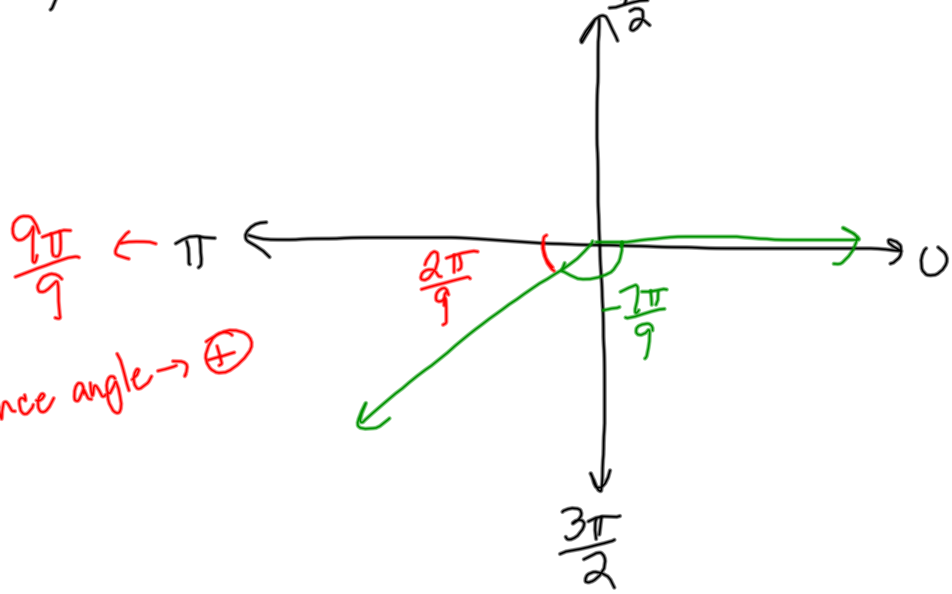
c. 310°

QIV

$360^\circ - 310^\circ = 50^\circ$



d. $-\frac{7\pi}{9}$



*reference angle → +

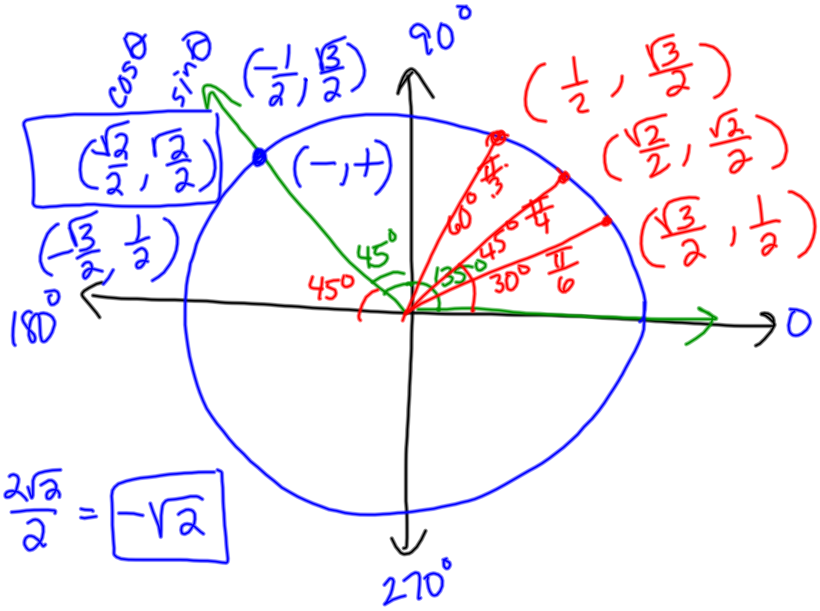
Evaluate the function without using a calculator.

a. sec 135°

$$\frac{1}{\cos 135^\circ}$$

$$\cos 135^\circ \to -\frac{\sqrt{2}}{2}$$

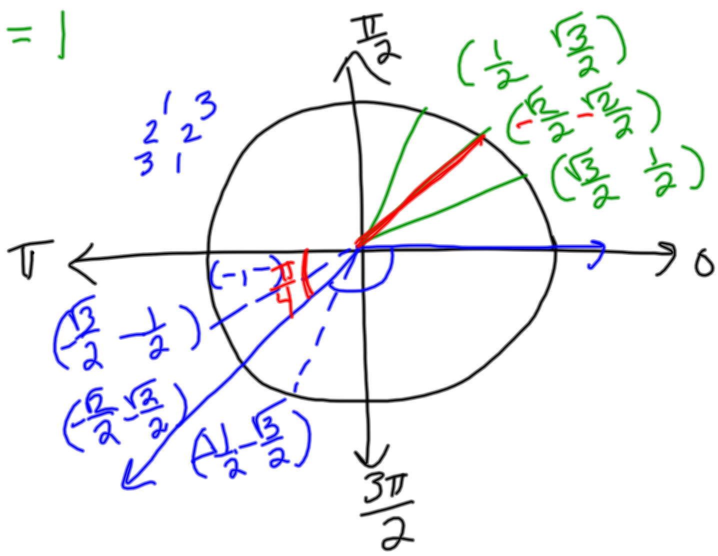
$$\sec 135^\circ = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$



b. $\tan\left(-\frac{3\pi}{4}\right) = 1$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$= \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$$



Estimate the horizontal distance traveled by a long jumper who jumps at an angle of 20° with an initial speed of 27 feet per second.

$$d = \frac{v^2}{32}\sin 2\theta$$

$$d = \frac{27^2}{32}\sin 2(20^\circ)$$

$$d = 14.64 \text{ feet}$$